Structure of two-dimensional mixed state of type-I superconductors

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It is shown that a two-dimensional-mixed-state layer produced on the inner surface of a hollow cylindrical sample carrying a sample not greatly exceeding the critical value has a macroscopic thickness and a periodic structure with a period of the order of the superconducting coherence length.

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The possibility of occurrence of a mixed state on the surface of a type-I semiconductor was discussed already in Landau's first papers[1–3] on the theory of the intermediate state. The discussion there concerned the state produced in a metal in the limit when the structure of the layered intermediate state is so strongly broken up that the distinction between the normal and superconducting phases becomes meaningless. Two different situations were considered. In one of them[1,2] the mixed state appeared under conditions of thermodynamic equilibrium as a result of an infinite branching of layers emerging to the surface. Following the studies by Shal'nikov and Meshkovskii[4] and by Lifshitz and Sharvin,[5] it became clear, however, that no branching sufficient for the formation of a mixed state takes place. In the other situation[3] the mixed state should occur on the internal surface of a hollow cylindrical superconductor through which a current $I > I_i$ flows, where $I_i = I_c(r_1^2 + r_2^2)/2r_1r_2$, $I_c = CH_c r_2/2$ is the Silsby critical current, and r_1 and r_2 are the radii of the inner and outer surfaces of the sample. In this situation, an electric field exists in the entire volume, but since the magnetic field on the inner surface is equal to zero, superconductivity should coexist with the electric field on this surface. The existence of such a two-dimensional mixed state was confirmed experimentally by Landau and Sharvin.[6] (For later investigations see[8,7]). Various opinions were advanced, concerning the actual structure of the two-dimensional mixed state. In[9,10], the two-dimensional mixed layer was regarded as a certain resistive state of the superconductor, homogeneous along the surface. Such a picture is justified in the case[10] of extremely strong electric fields, which practically suppress the superconductivity. In the usual case,[9] the role of the electric field remained unclear. Gor'kov and Dorokhov[11] have proposed a qualitative picture in which the two-dimensional mixed state consists of macroscopic superconducting sections without an electric field, separated by a normal phase.

In the present paper we obtain an exact solution of the problem of a hollow thin-wall cylinder with a current close to I_i. Near the inner surface there is produced a macroscopic layer of a peculiar intermediate state, whose structure has a period that decreases rapidly with increasing current. At $I - I_i \sim I_i$ the period becomes of the same order as the superconducting coherence length. On the other hand, the layer thickness remains macroscopic. Thus, we return in a certain sense to the initial picture of the mixed state.

The structure of the intermediate state produced near the internal surface is shown in Fig. 1. The z axis is the generatrix of the inner surface of the cylinder, and the x axis is radially directed into the sample. The superconducting regions are shown shaded. Assuming that the layer thickness is $x_0 \ll l = r_2 - r_1 \ll r \approx r_1$, we consider the planar problem. Let $z = \pm z_0(x)$, $(x < x_0)$ be the equations of the boundary curves OA and OB. In the normal phase located between them, the magnetic field $H \equiv H_y(x,z)$ can be sought in the form of an expansion, $H(x,z) = H_0(x) + (1/2)(\partial^2 H/\partial z^2)z^2$, in powers of z, since, as will be shown below, the period d of the structure is much smaller than x_0.

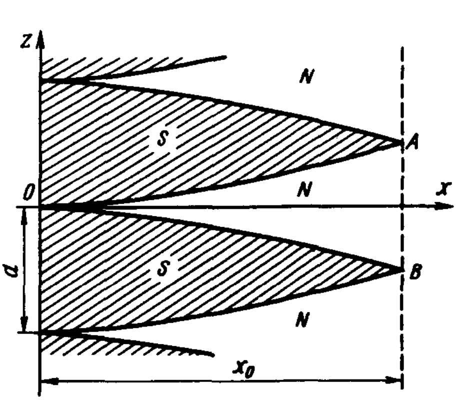

FIG. 1.

ssuming that the structure is static and therefore the magnetic field satisfied the aplace equation $\nabla^2 H=0$, we obtain

$$H(x,\ z) = H_0 - \frac{1}{2} H_0'' z^2, \tag{1}$$

here the prime denotes differentiation with respect to x. From (1) we obtain the on-zero components of the electric field

$$\frac{4\pi\sigma}{c} E_x = -\frac{\partial H}{\partial z} = H_0'' z, \qquad \frac{4\pi\sigma}{c} E_y = \partial H/\partial x \approx H_0 .$$

ere σ is the conductivity of the normal phase. We have neglected in these equations e terms connected with the curvature of the cylinder and whose relative order $_0^3/\Delta rd$ is assumed small. On the boundaries with the superconducting phase, i.e., at $=\pm z_0(x)$, the tangential components of the electric field should vanish. This yields e condition

$$\frac{d}{dx} (H_0' z_0) = 0. \tag{2}$$

distinguishing feature of the considered structure is that it is necessary to take into ccount the Laplace pressure in the boundary condition imposed on the absolute value f the magnetic field on the boundary with the superconducting phase, i.e., it is necesary to write $H^2/8\pi = H_c^2/8\pi - \alpha/R$, where H_c is the critical field, α is the surfaceension coefficient, and R is the curvature radius of the surface. Usually the term with he surface tension is vanishingly small. Here it plays the decisive role. Taking into ccount the inequality $x_0 \gg d$, the last condition can be represented in the form

$$H_0 = H_c\left(1 - \frac{\Delta}{2} z_0''\right), \tag{3}$$

here the surface tension is written, as usual, in the form $\alpha = (H_c^2/8\pi)\Delta$, where Δ is a ngth parameter of the same order as the coherence length $\xi(T)$. The conditions (2) nd (3) show that the function $z_0(x)$ must satisfy the equation $z_0 z_0''' = \mathrm{const}$, which has e following exact solution that satisfies, as we shall show, all the physical equirements:

$$z_0(x) = \frac{d}{2}\left(\frac{x}{x_0}\right)^{3/2}. \tag{4}$$

follows from (3) and (4) that the magnetic field in the normal phase is practically dependent (accurate to $\Delta d^3/x_0^4$) of z and its value at $x \ll x_0$ is

$$H(x) = H_c\left(1 - \frac{3\Delta d}{16 x_0^{3/2} x^{1/2}}\right). \tag{5}$$

he second term in the parentheses of the last formula should be regarded as a small correction to the first. This correction, being negative, tends to infinity as $x \to 0$, so that at very small x formula (5) cannot be used. This behavior of the magnetic field, however, agrees fully with the fact that it must vanish on the surface $x=0$. On the other boundary of the layer $x=x_0$, the magnetic field is less than H_c, but this decrease is relatively small $\Delta d/x_0^2 \ll 1$.

To determine completely the structure of the intermediate state, it remains to find the period d and the layer thickness x_0 by minimizing, at a given value of the total current through the sample, the thermodynamic potential $\mathcal{F}$, whose density[13,12] in the superconducting phase is equal to zero, and in the normal phase is $(H_c^2 - H^2)/8\pi$. The contribution of the region $x < x_0$ to the thermodynamic potential of a unit length of the cylinder is the sum of the volume part

$$\frac{1}{d}\int_0^{x_0} \frac{H_c^2 H^2}{8\pi} 2\pi r 2 z_0(x)\,dx = \frac{3}{64} H_c^2 \cdot \frac{\Delta d r}{x_0}$$

and the surface part

$$\frac{1}{d}\,\frac{H_c^2}{8\pi}\,\Delta 2\pi r 2 x_0 = \frac{H_c^2}{2}\,\frac{x_0 r \Delta}{2}\ .$$

The calculation of the contribution of the region $x > x_0$ coincides essentially with the corresponding calculations of[8]. We present the final results for d and x_0:

$$d = \frac{2 l_i \Delta}{I - I_i}\ , \qquad x_0 = \frac{1}{2}(3l)^{1/3}\left(\frac{l_i \Delta}{I - I_i}\right)^{2/3}. \qquad (6$$

The region of applicability of these formulas is defined by the inequalities

$$\frac{\Delta}{l} \ll \frac{I - I_i}{I_i} \ll 1, \quad \frac{l}{r} \ll \frac{I - I_i}{I_i}\ ,$$

where $r \approx r_1 \approx r_2$. By virtue of (6) they are equivalent to the use of the conditions $d \gg \Delta$, $x_0 \ll l$, $x_0^3 \ll \Delta r d$ in the derivation.

Formulas (6) enable us to trace the transition from the intermediate state to the mixed state with increasing current I. At the boundary of the applicability region, i.e., at $I - I_i \sim I_i$, we get from (6) $d \sim \Delta$ and $x_0 \sim \Delta^{2/3} l^{1/3}$. It can be stated that in this case the macroscopic metal layer $(x_0 \gg \Delta)$ adjacent to the inner surface is in the mixed state. The superconducting order parameter and the electric and magnetic fields in the mixed state are periodic functions of the coordinate along the surface, with a period on the order of the coherence length.

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