Asymptotic hadronic form factors in quantum chromodynamics

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We present a method that makes it possible to obtain exact (in all orders in the coupling constant) asymptotic expressions for the hadronic form factors in quantum chromodynamics. The method can be used also to calculate the form factors of bound states in other field-theory models, to calculate the hadronic large-angle scattering amplitudes, and so on. By way of example, the method is used to determine the exact asymptotic form of the pion electromagnetic form factor.

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We have previously[1] proposed a method of calculating the asymptotic hadronic form factors, and described results obtained when account is taken of the interaction at short distances in the lowest order in the coupling constant. The purpose of the present article is to present a method of summing the entire perturbation-theory series for this problem.

Consider, for example, the diagram of Fig. 1(a). We introduce an intermediate cutoff parameter σ such that $\mu_0^2 < \sigma^2 < |q^2|$, $\mu_0 \sim 1$ GeV. Integration over each loop is

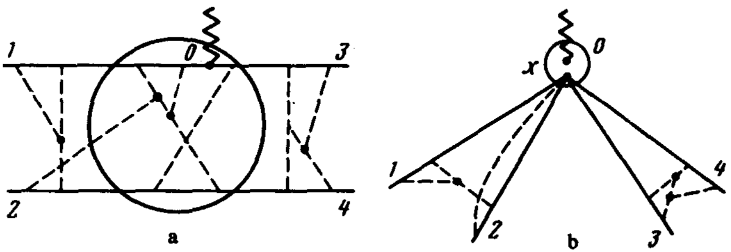

FIG. 1.

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broken up into a region of "large" ($(k_i > \sigma$) and "small" ($k_i < \sigma$) momenta. The circle on the diagram 1(a) determines one of the terms of the obtained sum, and separates the region of "small" distances (momenta $k_i > \sigma$ inside the circle) from the region of "large" distances (momenta $k_i < \sigma$ outside the circle). In the internal loops, the small momenta of the external loops can be neglected, and the diagram contracts effectively to that shown in Fig. 1(b), i.e., a local operator acts from the small region surrounding the point "0", and the non-contracted loops represent the part of its matrix element between the operator brackets). Summing different diagrams and different methods of breaking down one diagram, we obtain the expansion ($q = p' - p$, p' and p are the hadron momenta):

$$< p'| J(0)| p >_{\Lambda} \to \sum_n \oint dx\, C_n(\Lambda, x, \sigma) < p'| O_n(x) | p >_{\sigma} = \sum_n C_n(\Lambda, q, \sigma)$$

$$< p' | O_n(0) | p >_{\sigma} . \qquad (1)$$

The functions C_n in (1) represent the contribution of the contracted loops (region of small distances). The symbol σ in $\langle O_n \rangle_{\sigma}$ denotes that the integrals over the loops in this matrix element are cut off from above at σ (with an analogous general cutoff $\Lambda^2 \gg |q^2|$).

The expansion (1) is the intermediate stage of obtaining the solution, since the matrix element $\langle p' | O_n | p \rangle_{\sigma}$ contains a large momentum transfer. However, a direct examination of the diagrams that determine $\langle p' | O_n | p \rangle_{\sigma}$ shows the following[1] (assume for the sake of argument that quarks 1 and 2 on Fig. 1 belong to one hadron, while 3 and 4 belong to the other).

A) From among the diagrams containing particle exchanges between lines with large momentum transfer ($\sim q$), only diagrams of the eikonal type survive [Fig. 2(a)]. The remaining diagrams [of the type of Fig. 2(b)] are suppressed in power-law fashion (owing to the upper cutoff σ of the integrals).

B) Since the hadrons are neutral with respect to color, the eikonal contributions (cut off from above at σ and in the presence of the gluon mass λ) cancel each other completely in each order of perturbation theory.

As a result, the sum of all the $\langle p' | O_n | p \rangle_{\sigma}$ diagrams containing exchanges between lines with large momentum transfer is zero,[2] and we are left only with the diagrams that do not contain such exchanges [of the type of Fig. 1(b)]. This makes it possible to expand further in formula (1):

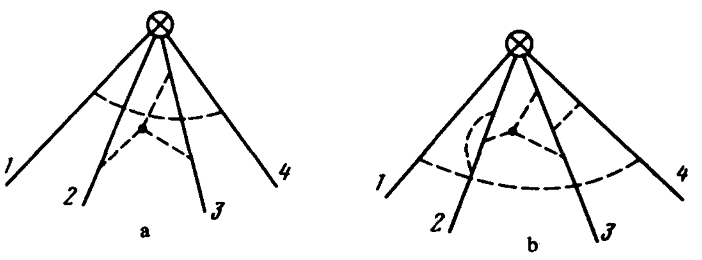

FIG. 2.

$$< p' | J(0) | p >_\Lambda \to \sum_{n_1} \sum_{n_2} C_{n_1 n_2}(\Lambda, q, \sigma) < p' | O_{n_1}(0) | 0 >_\sigma < 0 | O_{n_2}(0) | p >_\sigma, \quad (2)$$

We can now use the renormalization group, since the matrix element on the right-hand side of (2) no longer contain large invariant variables (exceeding σ). The renormalization group determines the dependence of $\langle O_n \rangle_\sigma$ on σ and of $\langle J \rangle_\Lambda$ on Λ (μ_0 is the subtraction point):

$$< O_n >_\sigma = Z_n^{-1}(\sigma^2/\mu_0^2, g^2) < O_n^R >_{\sigma=\infty}; \quad < J >_\Lambda = Z_J^{-1}(\Lambda^2/\mu_0^2, g^2) < J^R >_{\Lambda=\infty}.$$

The renormalization of $C_{n_1 n_2}$ in (2) therefore takes the form

$$< p' | J^R(0) | p > \to \sum_{n_1 n_2} C^R_{n_1 n_2}(q, \mu_0, g^2) < p' | O^R_{n_1}(0) | 0 > < 0 | O^R_{n_2}(0) | p >, \quad (3)$$

$$C^R_{n_1 n_2}(q, \mu_0, g^2) = Z_J(\Lambda^2/\mu_0^2, g^2) Z^{-1}_{n_1}(\sigma^2/\mu_0^2, g^2) Z^{-1}_{n_2}(\sigma^2/\mu_0^2, g^2) C_{n_1 n_2}(\Lambda, q, \sigma, g_0^2). \quad (4)$$

From (4) we obtain in the usual manner[2]

$$C^R_{n_1 n_2}(q, \mu_0, g^2) = C^R_{n_1 n_2}(\mu_0, \mu_0, \tilde{g}^2)(q^2/\mu_0^2)^d \exp\left\{ \int_{g^2}^{\tilde{g}^2} \frac{dS}{\beta(S)} [\gamma_J(S) - \gamma_{n_1}(S) - \gamma_{n_2}(S)] \right\}, \quad (5)$$

where $\tilde{g}$ is the effective coupling constant and γ_i are the anomalous dimensionalities [the factor $(q^2/\mu_0^2)^d$ maintains the usual dimensionality].

Formulas (3) and (5) constitute the main result of the present paper. For mesons, for example, the typical operators O_n [in (3) take the form $\partial^K(\bar{\psi} D^{n-k} \Gamma, \psi)$ ($\partial \equiv \partial/\partial x_\mu$, $D = i\partial - gB$, Γ is a numerical matrix]. If we neglect the terms $\sim g^2 \ln q^2$ in the functions C^R of (3) in comparison with unity, then the sums over the local operators O_{n_1} and O_{n_2} in (3) turn into matrix elements of bilocal operators, and we obtain the results of[1].

The asymptotic form (3) becomes greatly simplified in the region $g^2 \ln q^2 \gg 1$. Since the γ_n increase monotonically with increasing n, we can confine ourselves in the sums over n_1 and n_2 in (3) to the first terms of the expansion, i.e., to operators with minimal γ_i (more accurately, with minimal "twist"). Therefore at very large $|q^2|$ we have

$$< p' | J^R(0) | p > \to C^R_{12}(q, \mu_0, g^2) < p' | O^R_1(0) | 0 > < 0 | O^R_2(0) | p >, \quad (6)$$

where O_1 is the minimal-twist operator that yields a nonzero matrix element (O_2 is defined analogously), while the asymptotic form of C^R_{12} is determined from (5). In particular, for the pion form factor, the operator with the minimal twist is the axial current A^R_μ (and also its derivatives of the type $\partial_{\nu_1} \ldots \partial_{\nu_n} A^R_\mu$), and the asymptotic form is

$$\langle \pi^+(p')| J^R_\mu(0) | \pi^+(p)\rangle \to C^R_{\mu\nu\lambda}(q, \mu_0, g^2)\langle \pi^+(p') | A^{+R}_\nu(0)|0\rangle\langle 0| A^R_\lambda(0)| \pi^+($$

$$\to (p' + p)_\mu (f^2_\pi/q^2)\Omega^2 (N^2-1)N^{-2} (b_0 \ln q^2/16\pi^2)^{-1}, \quad (7$$

where J^R is the electromagnetic current, f_π is the pion decay constant ($\approx m_\pi$), $\Omega = 3/2$, $b_0 = (11/3N - 2/3M)$ for $SU(N)_c \otimes SU(M)_{fl}$ (the coefficient Ω^2 is the result o the contributions of the operators $\partial_{\nu_1} \ldots\ldots \partial_{\nu_n} A^R_\mu$). We note that as $q^2 \to -\infty$ the form factor approaches zero from below.[3)]

It should be noted that the method presented above for finding the asymptotic forms is in many respects schematic, so that a direct verification of the results is desirable. Such a verification was caried out by us for the pion form factor at the level of the leading logarithms of one- and two-loop diagrams (corrections $\sim g^2 \ln q^2$ and $g^4 \ln^2 q^2$). Namely, we compared the direct calculation with the diagrams with the expansion of the renormalization-group formulas (3) and (5) in powers of g^2. The results agreed.

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[1)]In the direct calculation of the diagrams we neglected the binding energy of the quarks in the hadron and introduced a gluon mass λ of the order of the reciprocal radius of the hadron. The quarks are located on (or near) the mass shell.

[2)]In electrodynamics, this statement can be easily verified in all orders in the coupling constants, since the eikonal contributions can be factored out and add up to an exponential. In chromodynamics we have verified this statement for one- and two-loop diagrams. It is clear from the analysis, however, that the cancellation will take place also in higher orders. In renormalized theories without vector mesons, there are no eikonal contributions at all, so that this statement is obviously valid. The contributions from eikonal diagrams that go from the region of large momentum are included in the functions C_n in (1). In nth order perturbation theory, the individual diagrams make contributions $\sim (g^2 \ln^2 q^2)^n$, but the sum of all the diagrams of nth order makes a contribution $\sim (g^2 \ln q^2)^n$, since all the doubly-logarithmic terms cancel out as a result of the neutrality of the hadrons with respect to color.

[3)]The asymptotic form of f_π is given by (7), since the anomalous dimensionalities of the electromagnetic and axial currents are equal to zero, and the Born approximation for the function C^R of (7) is $\sim O(g^2)$.

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