Spin waves in a degenerate solution of He^3 in superfluid He^4

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(Submitted November 3, 1976)
Pis'ma Zh. Eksp. Teor. Fiz. **25**, No. 1, 3–7 (5 January 1977)

The possibility of propagation of zero-sound and spin oscillations in a weak He^3-He^4 solution is investigated. It is shown that only one undamped mode, a longitudinal spin wave, exists.

PACS numbers: 67.60.Fp

The thermodynamic properties of a degenerate solution of He^3 in superfluid He^4 are completely determined by specifying the Fermi-liquid Landau function $f_{\vec{\sigma}\vec{\sigma}'}(\mathbf{p},\mathbf{p}')$.[1–3] The form of the function $f_{\vec{\sigma}\vec{\sigma}'}(\mathbf{p},\mathbf{p}')$, for solutions of arbitrary concentration (lower than the stratification concentration ~6%) is generally speaking unknown. Experiment yields only the first two harmonics of the Fermi-liquid function. In the case of a sufficiently low concentration, however, the bare He^3 particles dissolved in the superfluid background constitute a dilute degenerate Fermi gas of slow particles. The thermodynamic functions of this gas[4,5] correspond to those of a fermion system in which the two-particle interaction energy decreases quite rapidly at large distances. The properties of the solution can then be described by only one parameter, the s-scattering length a. An explicit expression for the function $f_{\vec{\sigma}\vec{\sigma}'}(\mathbf{p},\mathbf{p}')$ on the Fermi surface in a weakly-acting degenerate gas was obtained by Abrikosov and Khalatnikov[6]

$$f_{\vec{\sigma}\vec{\sigma}'}(\theta) = \frac{2\pi a\hbar^2}{m}\left[1 + 2\frac{p_0 a}{\pi\hbar}\left(2 + \frac{\cos\theta}{2\sin\frac{\theta}{2}}\ln\frac{1+\sin\frac{\theta}{2}}{1-\sin\frac{\theta}{2}}\right)\right]$$

$$-\frac{8\pi a\hbar^2}{m}\vec{\sigma}\vec{\sigma}'\left[1 + 2\frac{p_0 a}{\pi\hbar}\left(1 - \frac{1}{2}\sin\frac{\theta}{2}\ln\frac{1+\sin\frac{\theta}{2}}{1-\sin\frac{\theta}{2}}\right)\right], \quad (1)$$

where p_0 is the Fermi end-point momentum, and θ is the angle between the vectors $\mathbf{p}$ and $\mathbf{p}'$. With the aid of (1) we express the thermodynamic characteristics of the solution in terms of a. The energy spectrum of one He^3 atom placed in immobile superfluid He^4 is represented in the form

$$\epsilon = -\Delta + \frac{p^2}{2M}\left[1 - \beta\left(\frac{p}{p_c}\right)^2\right]. \quad (2)$$

Here $p_c = m_4 c$, m_4 is the mass of the He^4 atom and c is the speed of sound in pure He^4. The value of β was determined from measurements of the velocity of second sound in solutions and amounts to $\beta = 0.14 \pm 0.05$.[7] Using (1) and (2), and neglecting terms quadratic in the velocity $\mathbf{v}_s$ of the superfluid motion, we obtain the chemical potential of the He^3 in the superfluid solution

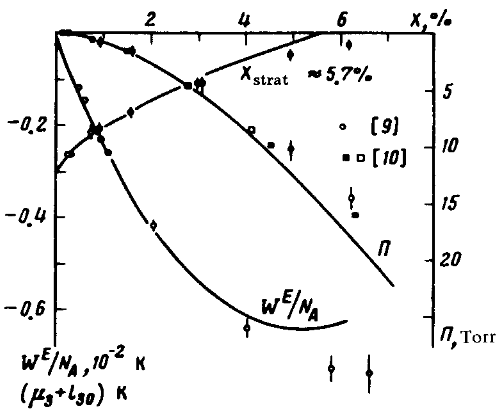

FIG. 1.

$$\mu_3 = -\Delta + \frac{p_0^2}{2M}\left[1 + \frac{4}{3}\frac{p_0 a}{\pi\hbar} + \frac{4}{15}\left(\frac{p_0 a}{\pi\hbar}\right)^2 (11 - 2\ln 2) - \beta\left(\frac{p_0}{p_c}\right)^2\right]. \qquad (3)$$

Calculating $\partial\mu_3/\partial p_0$ with allowance for the constancy of the number of particles, in analogy with[2,8], we obtain the total effective mass m^* of the excitations

$$\frac{m^*}{M} = 1 + \frac{8}{15}\left(\frac{p_0 a}{\pi\hbar}\right)^2 (7\ln 2 - 1) + 2\beta\left(\frac{p_0}{p_c}\right)^2. \qquad (4)$$

The osmotic pressure П in a system with a "supergap" is calculated from the condition that the chemical potentials of the solvent be equal on both sides of the "supergap," $\Pi = \int_0^{N_3} N_3(\partial\mu_3/\partial N_3)dN_3$ (N_3 is the number of He^3 atoms per unit volume of the solution). We define the excess enthalpy W^E of the system by the relation $W = -l_{30}N_3 - l_{40}N_4 + W^E$, where $-l_{30}$ and $-l_{40}$ are the latent heats of evaporation per atom of the pure He^3 and He^4 at $T = 0$, and W is the enthalpy per unit volume of the solution. It is easy to show that $W^E = (\mu_3 + l_{30})N_3 - \Pi$. The magnetic susceptibility of the spin system is also calculated in standard fashion:

$$\frac{\chi_{id}}{\chi} = 1 - 2\frac{p_0 a}{\pi\hbar} + \frac{8}{3}\left(\frac{p_0 a}{\pi\hbar}\right)(\ln 2 - 1). \qquad (5)$$

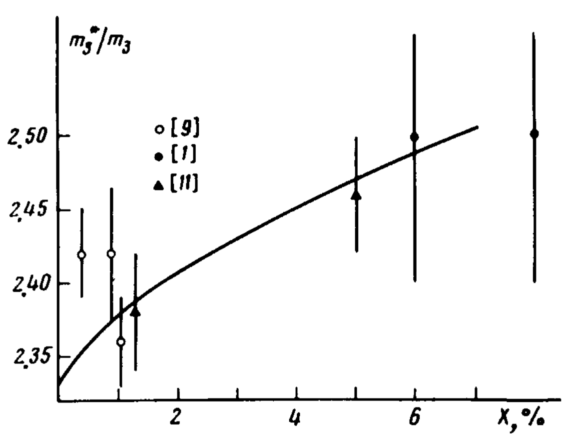

FIG. 2.

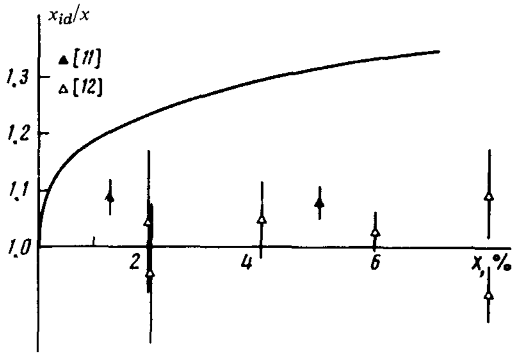

FIG. 3.

Figures 1—3 show comparisons of the curves calculated from formulas (3)—(5) with the experimental data of[1,9—12] for the values $a = -1.7$ Å, $M = 2.33 m_3$ ($x = N_3/(N_3+N_4)$. m_3 is the mass of the He^3 atom). The comparisons allow us to conclude that the function $f_{\sigma\sigma'}(\theta)$ in the form (1) describes the solution adequately up to concentrations on the order of 3—4%. At higher concentrations, the agreement is only qualitative. Since $a < 0$, corresponding to attraction between the impurity atoms, our calculations are suitable in the temperature range $T_c < T < T_0$, where T_0 is the degeneracy temperature and $T_c \approx (\gamma/\pi)(2/e)^{7/3} T_0 \exp[\pi\hbar/2p_0 a]$ is the temperature at which the structure of the excitation spectrum is altered by formation of Cooper pairs. [13] We have neglected retardation effects, since their contribution to function $f_{\sigma\sigma'}(\theta)$ is of the order of $(p_0/p_c)^2$.

We now use the function (1) for the study of high-frequency spin and zero-sound oscillations in the solution. Examination of the linearized system made up of the continuity and superfluid-motion equations and of the collisionless kinetic equation shows, in analogy with, [2,6] that an undamped longitudinal spin wave can propagate in the solution, with a velocity u given by

$$\frac{u}{v_0} = 1 + \exp\left\{\frac{\pi\hbar}{p_0 a} - 2\right\}, \qquad (6)$$

where v_0 is the Fermi velocity. The absorption coefficient of the high-frequency wave is equal to $1/\omega\tau$, where ω is the frequrncy of the wave and $\tau \approx (1/4\pi a^2 N_3 v_0)(T_0/T)^2$ is the relaxation time. The condition that the damping be small is then expressed in the form $T \ll (\hbar/|a|)(\hbar\omega/M)^{1/2}$. Substituting numerical values, we obtain $T \ll 4.5\times 10^{-6}\omega^{1/2}$. For comparison, in pure degenerate He^3 the analogous condition for zero sound takes the form $T \ll 1.2\times 10^{-6}\omega^{1/2}$. On the other hand, from the conditions $T > T_c$ and $\hbar\omega \ll T_0$ we obtain the frequency region in which spin wave observation is possible, $x^{2/3} \gg \omega/\omega_0 > (aN^{1/3})^2 x^{4/3} \times \exp[(\pi/3Nx)(1/a)]$, where $\omega_0 = (\hbar/M)N^{2/3}$ and $N = N_3 + N_4$. Substitution of the numerical values yields $x^{2/3} \gg 10^{-11}\,\omega > x^{4/3}\exp(-2.1/x^{1/3})$. An analysis of the equations also shows that undamped zero-sound and transverse ($m \neq 0$) spin oscillations cannot propagate in the solution.

I am grateful to A. F. Andreev for attentive guidance and very useful discussions, and to Z. V. Gausman and E. S. Gurova for help with the numerical calculations.

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