Effect of modulation instability on the relaxation of a relativistic electron beam in a plasma

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The spectra of Langmuir oscillations excited by a relativistic electron beam are singular in k-space. The influence of the modulation instability of such spectra on the relaxation of the electron beam is studied. It is shown that the instability-induced broadening of the oscillation spectrum decreases the energy flux into the plasma, lowers the turbulence level, and increases the relaxation length.

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1. The instability produced in an isothermal plasma by a relativistic electron beam is limited by the induced scattering from the ions. The rate of beam relaxation is determined by the spectrum of the resultant Langmuir turbulence. To describe the relaxation it is customary to use the weak-turbulence approximation (see, e.g., [1]). An analytic and numerical investigation of weak-turbulence spectra shows that they have a singular "jet-like"[2] character, i.e., they are located in k-space along the surfaces or lines (of two-dimensional and one-dimensional jets) or even at discrete points, the transverse width of the spectral distributions being very small. The energy of the oscillations is carried along the jets into the region of low frequencies: if the energy reaches the minimal frequency, then at $k=0$ a discrete component ("Langmuir condensate") is produced, the intensity of which may turn out to be quite high within the framework of the theory of weak turbulence.

It has already been noted[3] that the relaxation of a relativistic electron beam can be strongly influenced by the modulation instability of the Langmuir spectra. The reference there was either to instability of the condensate,[3,4] or to a strong instability of the spectrum,[5] which sets in at high beam intensities beyond the limits of applicability of the weak-turbulence description.

In the present article we call attention to the fact that an important role is played in the relaxation of a relativistic beam by the modulation instability of the jet spectra, which takes place even at low intensities.

A detailed investigation of weakly-turbulent spectra shows[6] that the jets have a fine structure and consist either of a set of discrete monochromatic waves, with a characteristic distance in k-space $k_{\text{dif}} \sim (1/r_D)\sqrt{m/M}$, or of diffuse spectral spots with characteristic dimensions k_{dif}. As shown in[7], the singularity

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of the weakly-turbulent spectra leads to the appearance of modulation instability. Its growth rate is maximal for jets with the fine structure described above, and its characteristic value γ_{mod} is given by

$$\frac{\gamma_{mod}}{\omega_p} \approx \frac{k_{dif}}{k} \frac{W}{nT} \tag{1}$$

(here k is the characteristic wave number, it is assumed that $k \gg k_{dif}$, and W/nT is the relative intensity of the oscillations in the jet). The development of modulation instability of one-dimentionsal jets and of a discrete component leads to a collapse of the Langmuir wave, which produces the effective damping[6]

$$\frac{\gamma_{eff}}{\omega_p} \approx \left(\frac{W}{nT} \frac{k_{dif}}{k}\right)^{3/2} \left(\frac{M}{m}\right)^{1/2} . \tag{2}$$

This damping usually turns out to be larger than the collisions damping and leads to a strong suppression of the one-dimensional and discrete components of the spectrum, so that the entire energy turns out to be concentrated in the two-dimensional jets. As shown in[8,9], in most cases the energy is absorbed as a result of the collapse before it reaches the condensate, and the condensate is simply not produced. The modulation instability of two-dimensional jets has a one-dimensional character (only oscillations transverse to the jet are unstable), and therefore leads not to a collapse, but only to a broadening of the spectrum in the transverse direction. It is this broadening which weakens the energy transfer from the beam to the plasma. We note that it is precisely the absence of condensates from our scheme which leads to a strong difference between our results and those of[4].

2. Let the angle width of the beam be $\Delta\theta > \Delta\epsilon/\gamma\epsilon$, where γ is the relativistic factor and $\Delta\epsilon$ is the energy spread in the beam. Then the growth rate of the beam instability is of the order of

$$\frac{\gamma_p}{\omega_p} \approx \frac{n'}{n} \frac{1}{\gamma(\Delta\theta)^2}$$

where n' and n are the electron densities in the beam and in the plasma, respectively. We assume that

$$\frac{m}{M} > \frac{\gamma_p}{\omega_p} > \frac{\nu_{ei}}{\omega_p} \frac{k}{k_{dif}} ,$$

where ν_{ei} is the collision frequency and $k \sim \omega_p/c$. Under this condition we can use the weak-turbulence approximation and neglect the Coulomb collisions. The principal energy of the Langmuir turbulence is concentrated in the two-dimensional jets—on a surface of revolution in the zone of the maximum growth rate, and is of the order of

$$\frac{W}{nT} \approx \frac{\gamma_p}{\omega_p}\left(\frac{k}{k_{dif}}\right)^2 . \tag{3}$$

The modulation instability increases the thickness of the jet to a value

$$(\delta k r_D)^2 \approx \frac{W}{nT} \frac{k_{dif}}{k} \approx \frac{\gamma_p}{\omega_p} \frac{k}{k_{dif}} . \tag{4}$$

The growth rate of the beam instability consists of symmetrically disposed instability and damping zones, with characteristic scale

$$\Delta k \sim k\Delta\theta. \tag{5}$$

So long as $\delta k < \Delta k$, the modulation instability exerts no influence on the beam relaxation. This corresponds to growth rates $\gamma_p < \gamma_{crit}$, where

$$\frac{\gamma_{crit}}{\omega_p} \sim (kk_{dif} r_D^2)(\Delta\theta)^2 \sim \frac{T}{mc^2}\sqrt{\frac{m}{M}}(\Delta\theta)^2 << \frac{m}{M}. \tag{6}$$

However, if $\gamma_p > \gamma_{crit}$, the modulation instability broadens a flat jet to a value exceeding the scale of the beam instability. In this case certain spectral components draw energy from the beam, and others, on the contrary, deliver energy to the beam, the total energy transfer to the plasma decreasing by a factor $(\Delta k/\delta k)^2$. For the energy density in the beam we now obtain

$$\frac{W}{nT} \sim \left(\frac{\gamma_p \gamma_{crit}}{\omega_p^2}\right)^{1/2}\left(\frac{k}{k_{dif}}\right)^2. \tag{7}$$

The local relaxation length—the distance over which the angular dimension of the beam doubles, increases in this case by a factor γ_p/γ_{crit}

$$l \sim l_o\left(\frac{\gamma_p}{\gamma_{crit}}\right)^{1/2}; \qquad l_o \approx \frac{\omega_p}{c}\frac{n'}{n}\frac{M}{m}\left(\frac{T}{mc^2}\right)^2\frac{1}{(\Delta\theta)^5}\frac{1}{\gamma^3}. \tag{8}$$

l_0 is the local relaxation length without allowance for the modulation instability.[1]

Formula (7) is valid up to growth rates

$$\gamma_p \sim \gamma_{crit}\left(\frac{k}{k_{dif}}\right)^2.$$

At larger growth rates, the beam instability will influence the character of the modulation instability, and at $(W/nT) > (kr_D)^2$ the spectrum begins to collapse as a unit; qualitatively, the weakening of the beam instability remains in effect.

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