Interband transitions and the possibility of current states in systems with electron-hole pairing

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Interband transitions make states with a homogeneous superfluid flux of particles impossible. At the same time, if the magnitude T_{ab} of the interband transitions does not exceed a critical value T_{ab}, then superfluid states with a coordinate-dependent particle flux and nonzero average flux become possible (this is attainable for spatially separated electrons and holes).

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A number of systems are discussed in the literature, in which pairing of electrons e with holes h can be realized via their Coulomb attraction: 1—a homogeneous semimetal that goes over under e–h pairing into the state of an "excitonic dielectric"[1]; 2—dielectric e–h drops in semiconductors[3]; 3—a system of superconducting or semimetallic films (filaments) with the electrons of one film paired with the holes of its neighbor.[3,4] The last system is of particular interest in view of the recently proposed mechanism of electron-hole superconductivity.[4] The restructuring of the ground state in the foregoing systems is accompanied by the appearance of an order parameter $\Delta = |\Delta| e^{i\phi}$ (the quantity $|\Delta|$ is proportional to the resultant gap in the elementary-excitation spectrum).

If no account is taken of interband transitions, the system energy does not depend on the absolute value of the phase ϕ. This corresponds to the existence of states with a homogeneous flux of particles. However, as shown in[5] for an excitonic dielectric, the interband transitions fix the phase ϕ. A similar role is played by the transitions between the bands of the pairing quasiparticles in the system 3, so that states with a homogeneous flux become impossible at finite magnitudes of the transition (see[4]). Incidentally, in the system 3, in contrast to the first two, these transitions are tunnel processes that become exponentially weaker with increasing distance between films, and can be made negligibly weak. As shown below, if the magnitude T_{ab} of the interband transitions does not exceed a certain critical value $\tilde{T}_{ab}$, it becomes possible to obtain in the system 3 states with nonzero average particle flux (see footnote 4). Prior to the averaging, the particle velocity depends on the coordinates. This dependence becomes increasingly weaker with decreasing T_{ab}.

At $T_{ab} \ll \tilde{T}_{ab}$ the barriers that separate the inhomogeneous (non-vortical) current states[1)] from one another turns out to be quite large (see[6] and the bibliography therein), so that these states can be metastable (superfluid), just as homogeneous current states in systems without interband transitions.

The Hamiltonian of the system is given by

$$H = \int \Psi_a^+(\mathbf{r})\epsilon_a(\hat{\mathbf{p}})\Psi_a(\mathbf{r})\,d\mathbf{r} + \int \Psi_b^+(\mathbf{r})\epsilon_b(\hat{\mathbf{p}})\Psi_b(\mathbf{r})\,d\mathbf{r}$$

$$+ V\int \Psi_b^+(\mathbf{r})\Psi_a^+(\mathbf{r})\Psi_a(\mathbf{r})\Psi_b(\mathbf{r})\,d\mathbf{r} + \int (T_{ab}\Psi_a^+(\mathbf{r})\Psi_b(\mathbf{r}) + \text{h.c.})\,d\mathbf{r}, \qquad (1)$$

where Ψ_{ab} is the electron-annihilation operator in the band a (valence) and b (conduction), respectively; $\epsilon_{a,b}(\mathrm{p}) = \mp(p^2 - p_0^2)(2m_{a,b})^{-1}$; V is the pairing interaction. The last term describes the interband transitions.[2)] From the system of Gor'kov's equations[3)], corresponding to the Hamiltonian (1), we easily obtain an equation in closed form for the anomalous function $F(1,2) = -i \langle T(\Psi_a(1)\Psi_b^+(2))\rangle$

$$\left[i\frac{\partial}{\partial t_1} - \epsilon_b(\hat{\mathbf{p}}_1)\right]\frac{1}{\Delta(1)}\left[i\frac{\partial}{\partial t_1} - \epsilon_a(\hat{\mathbf{p}}_1)\right]F(1,2) = -\delta(1-2) + \Delta^*(1)F(1,2), \qquad (2)$$

$$\Delta(1) = -T_{ab} - iVF(1,1). \qquad (2a)$$

We change over to new variables: $t=t_1$, $\mathbf{R}=\mathbf{r}_1$, $\tau=t_1-t_2$, $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$. Being interested henceforth in weakly inhomogeneous solutions, we shall assume that the characteristic length and the time of variation of the function $\Delta(R,t)$ are large in comparison with $\lambda_{\text{coh}} = v_F/|\Delta|$ (v_F is the Fermi velocity) and $|\Delta|$ respectively. Neglecting the derivatives of higher order in R and t in (2) and (2a), we obtain an equation for the phase $\phi(R,t)$:

$$u^2\nabla^2\phi - \frac{\partial^2\phi}{\partial t^2} = -\frac{4\Delta_0 T_{ab}}{NV}\sin\phi, \qquad (3)$$

where $N = 2m_a m_b nd/(m_a + m_b)p_0^2$ is the density of states on the Fermi surface (n is the particle density, d is the dimensionality of the system), and $u^2 = p_0^2(m_a m_b d)^{-1} \sim v_F^2$. In the derivation of (3) it was assumed also that $|1 - |\Delta|/\Delta_0| \ll 1$ (Δ_0 is the value of Δ in the homogeneous case). It can be shown that this is ensured by the inequality

$$T_{ab}(VN\Delta_0)^{-1} \ll 1, \qquad (4)$$

which we shall assume to hold.

In the absence of interband transitions ($T_{ab}=0$), Eq. (3) coincides formally with the equation for ordinary superconductors.[7] It describes both the propagation of sound (waves of quasiparticle density variation($\delta n_{a,b} \sim \partial\phi/\partial t$)) with a dispersion law $\omega(k) = uk$. It describes also the motion of the system as a whole with fixed density $n_{a,b}$. The latter means independence of ϕ of t, so that at $T_{ab} = 0$ the corresponding solutions of (3) take the form $\phi(\mathbf{R}) = \vec{\mathcal{P}}\mathbf{R}$ and describe a homogeneous flux of particles with velocity $v = \nabla'\phi/(m_a + m_b) = \mathcal{P}/(m_a + m_b)$. In the presence of interband transitions ($T_{ab} \neq 0$), however, the collective-excitation spectrum is altered and does not have an acoustic character (see also[5]). By linearizing (3) we obtain $\omega(k) = [u^2k^2 + 4\Delta_0 T_{ab}/NV]^{1/2}$ —an optical mode is produced.[4)] Changes take place also in the solutions $\phi = \phi(R)$ which describe the macroscopic flux of particles. At $T_{ab} \neq 0$ Eq. 3 has no solutions $\phi(\mathbf{R}) = \vec{\mathcal{P}}\mathbf{R}$, and homogeneous current states in the system are impossible. We consider one-dimensional motion in an aribitrary direction. Equation (3) then coincides with the equation for the physical pendulum. Its solutions correspond to two possible pendulum motions: I—periodic, when the pendulum does not reach the upper point; II—rotational. Thus, if one excites in the system motion with intitial velocity $v < \tilde{v}$, where

$$\tilde{v} = \frac{4}{(m_a + m_b)}\left(\frac{T_{ab}\Delta_0}{NVu^2}\right)^{1/2}, \tag{5}$$

then the dependence of the phase ϕ on the coordinates has a periodic character and the particle flux $j \sim \nabla\phi$ vanishes when averaged over the period of the oscillations. On the other hand, if the initial velocity is $v > \tilde{v}$, then the phase ϕ depends monotonically on R, and the average value of the flux differs from zero (its value prior to averaging depends on R). However, the velocity v must not exceed the critical pair-breaking velocity $v_{cr} = 2\Delta\sqrt{m_a m_b}/(m_a + m_b)p_0$ (see[4]), for otherwise the system goes over into the normal state with $\Delta \equiv 0$. The inequality $\tilde{v} < v < v_{cr}$ leads to the following condition for the possible existence of states with nonzero average superfluid particle flux in the system:

$$T_{ab} < \tilde{T}_{ab} = \frac{1}{4d} NV\Delta_0 . \tag{6}$$

We note that the condition (4) for the applicability of the initial equation (3) is equivalent to the inequality $T_{ab} \ll \tilde{T}_{ab}$. Therefore, if (4) is valid, states with non zero average superfluid flux are certainly possible in the system (the initial velocity of this motion should exceed $\tilde{v}$ (5)).[5)]

For system 3, at the parameter values chosen in[4], namely $m_e = m_h = m^* = 0.03m_0$, $\epsilon = 3$, $p_0^{-1} \sim D \sim a^* \sim 50$ Å (D is the distance between the films, $a^* = \epsilon/(m^*e^2)$) and $W = 4$ eV (W is the work function of the electron going from the film into the separating layer) we obtain: $T_{ab} \sim m^*e^4/\epsilon^2$, $\tilde{T}_{ab} \sim (m^*e^4/\epsilon^2) \times \exp(-D\sqrt{2m_0W}) \sim (m^*e^4/\epsilon^2)e^{-60}$. The inequality $T_{ab} \ll \tilde{T}_{ab}$ is satisfied in this case with a tremendous margin, so that superfluid current states exist, and the minimum initial velocity needed for their excitation is vanishingly small: $\tilde{v} \sim 10^{-22}$ cm/sec. At such small T_{ab}, all the states are practically indistinguishable from states with a homogeneous particle flux. Indeed, the flux modulation length $L_T = (NVu^2/4T_{ab}\Delta_0)^{1/2}$ is astronomically large: $L_T \sim 10^{24}$ cm.

On the other hand, when D or W decreases, T_{ab} increases and modulation of the flux, as well as states with an oscillating flux, can become observable.

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[1)]They correspond to different integer values $n=(2\pi)^{-1}\oint\nabla\phi\, dr$ in the case when the system is not singly-connected, for example, in a system 3 in the form of coaxial cylindrical layers with e and h conductivities.

[2)]Interband transitions correspond also to other terms not written out in (1), for example $\int dr(\Psi_b^+\Psi_b^+ V\Psi_a\Psi_b + \text{h.c.})$ etc. We confine ourselves here only to an analysis of the simplest case.

[3)]The analysis is carried out for the temperature $T=0$, and covers by the same token also the quasi-two-dimensional systems 3, to which for example, the Ginzburg—Landau theory is applicable (owing to the divergence of the fluctuations at $T>0$).

[4)]At $T_{ab}\neq 0$, Eq. (3) has also more complicated solutions (corresponding to vortices or solitons). É.B. Sonin has kindly informed us that he has arrived at similar conclusions by considering the Ginzburg—Landau equations for an excitonic dielectric at temperatures close to critical.[6]

[5)]On the other hand, a wider range of values of T_{ab} that admit of the existence of current states is defined by inequality (6) only in order of magnitude.

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