Anomalies of low-temperature resistivity of metals

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We investigated the temperature dependence of the resistivity of a metal under conditions when the wavelength of the thermal phonon exceeds the impurity mean free path of the electron. It is shown that, starting with $T=0$, the resistivity first decreases with decreasing temperature like $\Delta\rho(T)\sim -T^2$ and then, going through a minimum, begins to grow logarithmically in the region $(\Theta/\epsilon_F)^2<\tau T<\Theta/\epsilon_F$.

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In the analysis of the resistivity of metals it is customary to use the classical Boltzmann equation. This approach can be used for scattering by phonons only when the characteristic change of the electron momentum by the scattering is much larger than its uncertainty. This condition is violated if the wavelength of the thermal phonon is not small in comparison with the impurity path length of the electron. In this case the interference between the phonon and the impurity scattering mechanisms should determine to an appreciable degree the temperature dependence of the kinetic coefficients. The corresponding temperature region is specified by the condition $T\lesssim u/\tau v = T_{qu}$ (u is the speed of sound, v is the Fermi velocity, and τ is the impurity scattering time). For impurities with a Coulomb potential $T_{qu}\sim c\Theta$ (c is the impurity concentration and Θ is the Debye temperature) reaches several degrees at $c\sim 0.01$, while in metals with large Debye temperature (say, beryllium), it reaches even several dozen degrees. We consider the phonon contribution to the resistance of metals with nonmagnetic impurities at $T\lesssim T_{qu}$. On the one hand, the Boltzmann equation does not hold in this temperature region and a quantum kinetic equation must be used to solve the problem. On the other hand, it is important to determine the physical mechanism that leads to interference between the elastic and inelastic collisions. One such mechanism is the increase of the time of interaction between the electron and the phonon as a result of the effective slowing down of the electrons that diffuse in the field of randomly located effects. To clarify this concept, let us calculate the sum of ladder diagrams shown in Fig. 1 (a detailed summation was carried out in[1] for scattering by a photon). We confine ourselves for simplicity to s-scattering by impurities. At $T=0$, this summation leads to the following expression for the electron-phonon interaction amplitude:

$$\tilde{g}(\mathbf{q}) = g(\mathbf{q})\left\{1+\frac{\gamma}{1-\gamma}\,\Theta\!\left(\frac{1}{2}|\omega| - |\epsilon|\right)\right\},\qquad \gamma=\frac{\operatorname{arc\,tg}\tau q v}{\tau q v}. \tag{1}$$

Here $g(\mathbf{q})$ is the bare electron-phonon amplitude. At $\tau qv\ll 1$ and $|\epsilon|<|\omega/2|$ we have $\tilde{g}(\mathbf{q})=3g(\mathbf{q})/(\tau qv)^2$, i.e., $\tilde{g}(\mathbf{q})$ increases anomalously in the case of small momentum transfers. This behavior plays a decisive role in the tem-

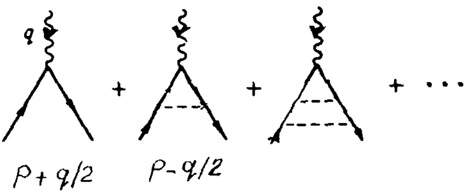

FIG. 1.

perature dependence of the kinetic coefficients at $T \lesssim T_{qu}$, when smallness of the phonon momenta is ensured by the low temperature.

To calculate the resistance we have used a quantum kinetic equation derived with the aid of the Keldysh diagram technique.[2] It can be shown that for electrons in nonmagnetic metals, the exact quantum equation in the case of a homogeneous electric field E is of the form

$$-(\mathbf{E}\mathbf{v})\frac{\partial n}{\partial \epsilon}\frac{\partial}{\partial \epsilon_{\mathbf{p}}}\left(\eta A + 2\operatorname{arc\,tg}\frac{2\eta}{\Gamma}\right) = \Sigma^{+}G^{-} - \Sigma^{-}G^{+}, \tag{2}$$

$$n = (e^{\beta\epsilon}+1)^{-1}, \quad \eta = \epsilon - \epsilon_{\mathbf{p}} - \mathrm{Re}\,\Sigma_r(\epsilon, \mathbf{p}), \quad \beta = 1/T, \quad \Gamma = -2\mathrm{Im}\Sigma_r(\epsilon, \mathbf{p}),$$
$$A = -2\Gamma/(\eta^2 + \Gamma^2/4).$$

$G^{\pm}$ are the "kinetic" Green's functions [at equilibrium we have $G^{\pm} = \pm in(\pm\epsilon)A$], and $\Sigma^{\pm}$ are the corresponding mass operators. From the structure of the quantum collision integral $I = \Sigma^{+}G^{-} - \Sigma^{-}G^{+}$ it follows that the nonequilibrium parts δG of all four Green's functions are the same (in the approximation linear in the field). Since the impurity part of the resistance is small in comparison with the phonon part at $T \lesssim T_{qu}$, Eq. (2) can be solved by iteration in terms of the electron-phonon interaction. Assuming s-scattering by the impurities, we represent the collision integral in the form $-i\delta G/\tau + I_{ph}\{\delta G_0\}$. The quantity $\delta G_0 = i(\mathbf{E}\mathbf{v})(\partial n/\partial\epsilon)A^2/2$ was obtained with the phonon mechanism of scattering neglected. Using the expression for the current $\mathbf{j} = 2i\int \mathbf{v}\delta G d^4p/(2\pi)^4$, we obtain a formula for the phonon contribution to the resistivity

$$\Delta\rho/\rho_0 = -\frac{2m}{N_e}\int\frac{d^4\mathbf{p}}{(2\pi)^4}\frac{(\mathbf{E}\mathbf{v})}{E^2} I_{ph}\{\delta G_0\}, \qquad \rho_0 = m/\tau N_e. \tag{3}$$

Figure 2 shows different contributions to the integral (3). The solid lines correspond to the matrix Green's function (the smooth lines correspond to electrons and the wavy ones to phonons). The circle marks the matrix corresponding to external Green's functions in the collision integral. The shaded triangles represent ladders made up of dashed impurity lines. One electron line of each loop (in proper sequence, including those inside the ladders) is a nonequilibrium line, i.e., it is equal to δG_0. It turns out that in the quantum region ($T \lesssim T_{qu}$) the main contribution to $\Delta\rho$ comes from the fourth diagram, in which the nonequilibrium line is marked by a circle. The corresponding part of the collision integral is of the form $-i\Gamma_{ph}\delta G_0$. Thus, at $T \lesssim T_{qu}$ the τ approximation is valid for scattering by phonons. The reasons is that at $\tau q v \ll 1$

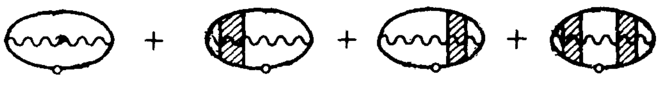

FIG. 2.

the inelastic process is accompanied by multiple scattering by impurities, as a result of which the momentum and energy relaxation processes become independent. The expression for the phonon damping of the electrons, which corresponds to the main singularities of the ladder diagrams, is of the form

$$\Gamma_{ph}(p) = - \int \frac{d^3 q}{(2\pi)^3} A(p+q) \frac{n(\epsilon) - n(\epsilon + \omega_{\mathbf{q}})}{2} \left(\frac{\zeta}{1-\zeta}\right)^2 | g(\mathbf{q}) |^2 ,$$

$$\zeta = \frac{\operatorname{arc\,tg} \tau (qv + \omega_{\mathbf{q}}^*) + \operatorname{arc\,tg} \tau (qv - \omega_{\mathbf{q}}^*)}{2\tau\, qv} + \frac{i}{4\tau\, qv} \ln \frac{\tau^2 (qv + \omega_{\mathbf{q}}^*)^2 + 1}{\tau^2 (qv \mp \omega_{\mathbf{q}}^*)^2 + 1} ,$$

$$\omega_{\mathbf{q}}^* = (1 + \lambda)\, \omega_{\mathbf{q}} \qquad (4)$$

λ is the electron-phonon coupling constant.

Substituting (4) in (3) and using the Debye approximation for the phonon spectrum, we obtain for the temperature-dependent part of the resistance in the region $T \lesssim T_{qu}$

$$\frac{\Delta\rho(T)}{\rho_o} = \frac{\lambda\nu}{(\tau\epsilon_F)^2} f(1/t), \quad \nu = \theta/\epsilon_F, \quad t = 2\pi\tau T/3(1+\lambda)\nu^2,$$

$$f(x) = 13.5[\psi(x) + 3x\psi'(x) + x^2\psi''(x) - \ln x - 2].$$

$\psi(x) = \Gamma'(x)/\Gamma(x)$ is the logarithmic derivative of the Euler Gamma function.

Figure 3 shows a plot of the function $f(1/t)$. As the temperature rises from absolute zero, the resistivity first decreases quadratically

$$\Delta\rho/\rho_o = (-9\pi^2/8)(\lambda\nu/(\tau\epsilon_F)^2)t^2, \qquad (t \ll 1)$$

goes through a minimum at $t = 2.0$, after which it increases and assumes in the region $1 \ll t \ll 1/\nu$ the logarithmic dependence

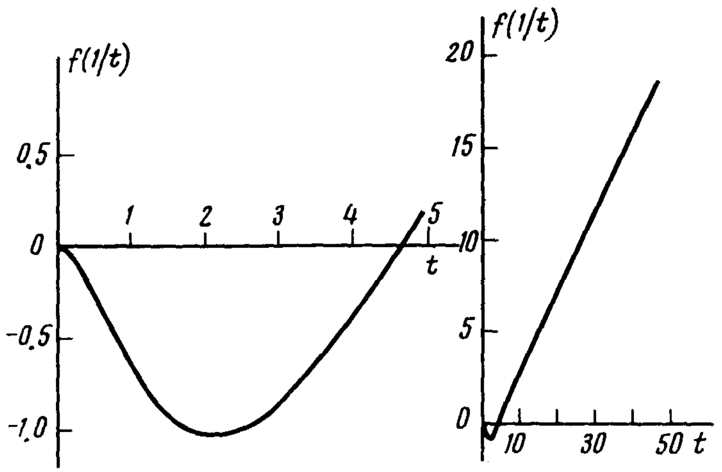

FIG. 3.

$$\Delta\rho/\rho_o = 13.5\,\lambda\nu/(\tau\epsilon_F)^2 \ln t/\gamma e^2 , \quad \gamma = 1.781 \ldots .$$

At $t \sim 1/\nu$ (i.e., $T \sim T_{qu}$) the contribution from the fourth diagram of Fig. 2 ceases to be the principal one, and at $t \gg 1/\nu$ the temperature dependence is determined by the first diagram, i.e., the classical Bloch law $\Delta\rho \sim T^5$ is restored.

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[1]A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskiĭ, Metody kvantovoĭ teorii polya v statisticheskoĭ fizike (Quantum Field-Theoretical Methods in Statistical Physics), Moscow, 1962.

[2]L. V. Keldysh, Zh. Eksp. Teor. Fiz. 47, 1515 (1964) [Sov. Phys. JETP 20, 1018 (1965)].