Scattering of phonons by Abrikosov vortex filaments in vanadium

N. A. Ped'ko and B. K. Chakal'skiĭ
A. F. Ioffe Physicotechnical Institute, Academy of Sciences of the USSR

(Submitted 18 June 1986)
Pis'ma Zh. Eksp. Teor. Fiz. **44**, No. 5, 224–226 (10 September 1986)

The phonon component of the thermal conductivity is clearly identified in vanadium ($\rho_{300\text{ K}}/\rho_{4.2\text{ K}} = 14.4$) in the superconducting state (1.8–3 K). The scattering of phonons by vortices is inferred from the magnetic-field dependence of the thermal conductivity in the mixed state near H_{c1} and the effective scattering cross section is estimated to be ($\sigma^{\perp} = 9.2\times 10^{-7}$ cm, $\sigma^{\parallel} = 4.5\times 10^{-7}$ cm).

We studied the thermal conductivity (κ) of vanadium (a type II superconductor with a weak electron-phonon coupling[1]) in the temperature interval 1.8–15 K. The measurements were carried out with two samples with a resistivity ratio $\rho_{300\text{ K}}/\rho_{4.2\text{ K}}$ of 44 and 14.4 and with residual resistivities of 0.49×10^{-6} $\Omega\cdot$cm and 1.6×10^{-6} $\Omega\cdot$cm. Sample 1 is a polycrystal $\phi = 1.4$ mm, $h = 40$ mm, $T_c = 5.38$ K, $\Delta_0 = 9.5$ K is the half-width of the energy gap ($T = 0$ K), and $l_0 = 1.17\times 10^{-5}$ cm is the mean free path of electrons. Sample 2 is a single crystal $4\times 4\times 40$ mm, $T_c = 5.06$ K, $\Delta_0 = 8.55$ K, and $l_0 = 3.6\times 10^{-6}$ cm.

1. The temperature dependence $\kappa(T)$ of samples 1 and 2 is shown in Fig. 1a. The electronic nature of $\kappa(T)$ in the normal state stems from the fact that the Lorentz number does not exceed the Sommerfeld value. The dependence $\kappa^n(T) \sim T$ suggests that electrons are scattered by the lattice impurities and defects. The thermal conductivity of sample 1 in the superconducting state (κ^s) is described well by the Geĭlikman[2] equation, which takes into account the scattering of electronic excitations by point defects (the dashed curve 1a in Fig. 1a):

$$\kappa^s = \kappa^n \frac{6}{\pi^2}\left\{\frac{b^2}{e^2+1} + 2\sum_{s=1}^{\infty}\frac{(-1)^{s+1}}{s}e^{-sb} + 2b\ln(1+e^{-b})\right\}, \qquad (1)$$

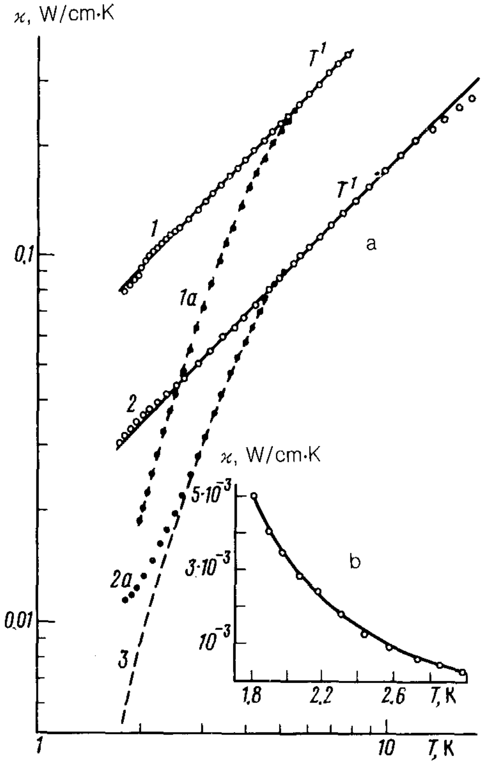

FIG. 1. (a) Temperature dependence of the thermal conductivity of vanadium for samples 1 and 2. Curves 1 and 2 correspond to the normal state and curves 1a and 2a correspond to the superconducting state. Curves 3 and 1a—calculations of the electronic thermal conductivity in the superconducting state on the basis of Eq. (1). (b) Temperature dependence of the phonon component of the thermal conductivity.

where $b = \Delta(T)/T = [f(T)/t]\Delta_0/T_c$, and $t = T/T_c$. We used the data for $f(t)$ taken from Ref. 3 to calculate $\Delta(T)$. In the case of sample 2, the electronic component of the thermal conductivity calculated from Eq. (1) for $\Delta_0/T_c = 1.692$ is in good agreement with the experiment at temperatures $T = (0.6–1)T_c$ (curve 3 in Fig. 1a). At $T < 0.6$ T_c the experimental value of κ^s is higher than the calculated value, which stems from the appearance of an appreciable phonon component ($\kappa_{\rm ph}$). The clearly identifiable $\kappa_{\rm ph}$ is shown in Fig. 1b.

2. The presence of the phonon component of thermal conductivity in addition to the electronic component at $T < 0.6$ T_c, and its absence at $T > 0.6T_c$ in sample 2, accounts for the different shape of the curve for the thermal conductivity versus the magnetic field in the mixed state. This situation is illustrated in Fig. 2. At temperatures $T > 0.6$ T_c, with the electronic component the dominant component in the total thermal conductivity, in the mixed state the $\kappa(H)$ curve has no dips near H_{C1} which are attributable to the scattering of normal excitations by Abrikosov vortex filaments (curves 1 and 2 in Fig. 2). Such dips were detected in pure vanadium samples[1] (in sample 1 to a lesser degree). In sample 2 the normal excitations are scattered principally by impurities and defects and in a pure sample they are scattered primarily by vortices. Thus, for example, in the case of scattering by impurities and defects, the thermal resistance of normal excitations determined experimentally at $T = 2.43$ K

increases from 0.7 cm·K/W for a pure sample[1] to 69 cm·K/W for sample 2. In the case of scattering by vortices, the thermal resistance of normal excitations, which does not depend on the sample purity, is ~2.2 cm·K/W.

3. At $T<0.6\ T_c$ the phonon component of thermal conductivity of the sample is comparable to the electronic component and in the mixed state the $\kappa(H)$ curve exhibits dips near H_{C1}, which are attributable to the scattering of phonons by Abrikosov vortex filaments (curves 3, 5, and 6 in Fig. 2). The thermal conductivity depends on the magnetic field in the following way. In the Meissner region ($H<H_{C1}$), $\kappa(H)$ does not depend on the field. At $H>H_{C1}$ the $\kappa(H)$ curve initially drops sharply, as the field is increased, because of the increase in the scattering of phonons by vortices. After it goes through a minimum, the $\kappa(H)$ curve begins to rise primarily because of the increase in the number of normal excitations as H approaches H_{C2}, where the electronic component is the dominant component in the total thermal conductivity. The minimum value of the thermal conductivity in the mixed state (with $\mathbf{H}\perp\nabla\mathbf{T}$; curves 3 and 5 in Fig. 2) coincides with the calculated value of the electronic component in the

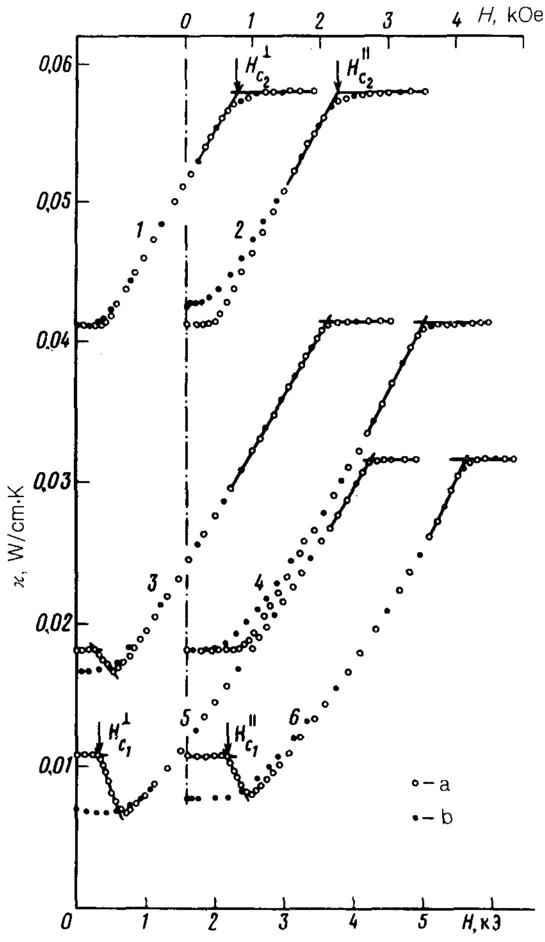

FIG. 2. Behavior of the thermal conductivity of sample 2 in a transverse magnetic field ($H\perp\nabla T$) (curves 1, 3, and 5) and in a longitudinal magnetic field ($H\parallel\nabla T$) (curves 2, 4, and 6), where curves 1 and 2 were plotted for $T/T_c=0.68$, curves 3 and 4, for $T/T_c=0.48$, and curves 5 and 6, for $T/T_c=0.364$. (a) Forward curve, (b) inverse curve.

superconducting state (curve 3 in Fig. 1a). This shows that the phonons are scattered primarily by vortices in a transverse magnetic field. The $\kappa(H)$ curve with dips exhibits a strong hysteresis (Fig. 2). For each orientation of the magnetic field the reciprocal of the $\kappa(H)$ curve does not return to the original value but instead remains at the level of the minimum value because of the freezing-in of the magnetic flux in the sample.

4. Since there is no theory which can describe the behavior of the phonon thermal conductivity in the mixed state near H_{C1}, we will estimate the cross section for scattering of phonons by vortices on the basis of the experimental data. In a magnetic field the total thermal conductivity near H_{C1} changes as a result of scattering of phonons by vortices:

$$\Delta\kappa(H) = \kappa_{\phi}(H) - \kappa^{s}_{ph}, \quad (2)$$

where $\kappa_{ph}(H)$ and κ^{s}_{ph} are the thermal conductivities of the phonon component in a magnetic field H and in the absence of H. We single out two frequencies of the scattering of phonons by vortices $(1/\tau_b)$ and by other contributions to the scattering $(1/\tau_0)$

$$1/\tau_{ph}(H) = 1/\tau_b + 1/\tau_0, \quad (3)$$

$$1/\tau_b = \sigma v N, \quad (4)$$

where σ is the scattering cross section, $v = \bar{v} = 3.2\times 10^5$ cm/s is the speed of sound in vanadium,[4] $N = \Delta B/\phi_0$ is the vortex density, and $\phi_0 = 2\times 10^{-7}$ Г·cm^2 is a fluxoid. Using (3) and (4), we substitute $\tau_{ph}(H)$ into Eq. (2):

$$\Delta\kappa(H) = \kappa_{ph}(H) - \kappa^{s}_{ph} = \frac{1}{3}Cv^2\tau_{ph}(H) - \frac{1}{3}Cv^2\tau_0 = \kappa^{s}_{ph}\left(\frac{1}{1+\sigma v N\tau_0} - 1\right), \quad (5)$$

where $C = \alpha T^3 = 2.6\times 10^5$ J/cm^3·K is the specific heat of the vanadium lattice[5] at $T = 1.84$ K. Using $\tau_0 = 3\kappa^{s}_{ph}/Cv^2$ and (5), we find the scattering cross section

$$\sigma = \frac{Cv}{3N\kappa^{s}_{ph}}\left(\frac{\kappa^{s}_{ph}}{\Delta\kappa(H) + \kappa^{s}_{ph}} - 1\right).$$

The cross sections for scattering of phonons by Abrikosov vortex filaments are $\sigma^{\perp} = 9.2\times 10^{-7}$ cm and $\sigma^{\parallel} = 4.5\times 10^{-7}$ cm. These values were determined from the experimental data at $T = 1.84$ K $\kappa^{s}_{ph} = 0.00492$ W/cm·K for $\mathbf{H}\perp\nabla\mathbf{T}$ [$\Delta\kappa(H) = -0.00345$ W/cm·K, $\Delta B = 300$ G] and for $\mathbf{H}\parallel\nabla\mathbf{T}$ [$\Delta\kappa(H) = -0.00175$ W/cm·K, $\Delta B = 140$ G].

A theoretical estimate of the cross section for scattering of phonons by vortices, which is based on the assumption that the wavelength of a phonon is shorter than the coherence length (ξ), yields $\sigma = 1/Nl_{ph} \simeq kT\xi^2/\hbar v_F \simeq 3\times 10^{-7}$ cm. Here $l_{ph} = l^{n}_{ph}/N\xi^2 \simeq \hbar v_F/kTN\xi^2$, where l^{n}_{ph} is the mean free path of phonons in a normal metal, and v_F is the Fermi electron velocity.

This estimate of the cross section for scattering of phonons by vortices is in good agreement with the estimate based on the experimental data. It is interesting that the cross section for scattering of normal excitations by vortices is [1] $\sim 6\times 10^{-7}$ cm.

We wish to thank Yu. M. Gal'perin for a discussion of the experimental results and for estimating the mean free path of phonons in the vortices.

[1]B. K. Chakal'skiĭ, N. A. Red'ko, S. S. Shalyt, and V. M. Azhazha, Zh. Eksp. Teor. Fiz. **75**, 1320 (1978) [Sov. Phys. JETP **48**, 665 (1978)].
[2]B. T. Geĭlikman, Zh. Eksp. Teor. Fiz. **34**, 1042 (1958) [Sov. Phys. JETP **7**, 721 (1958)].
[3]B. Mühlschlegel, Zs. Phys. **155**, 313 (1959).
[4]D. J. Bolef and M. Menes, J. Appl. Phys. **31**, 1426 (1960).
[5]R. Redebaugh and P. H. Keesom, Phys. Rev. **149**, 209 (1966).

Translated by S. J. Amoretty