# Temperature dependence of the density of $^4$He in narrow gaps near the $\lambda$-point

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Quantitative measurements have been carried out on the temperature dependence of the density of liquid $^4$He near the $\lambda$-point in a thin parallel-plane gap with a well-controlled geometry and a thickness $d \simeq 3.3 \times 10^{-5}$ cm. The results are compared with theoretical predictions.

An experimental study of the temperature dependence of the total density of helium in a film or gap can yield information on the behavior of the $\lambda$-transition under conditions of a bounded geometry.[1] In the present letter we report a quantitative study of the temperature dependence near the $\lambda$-point of the density difference $\Delta\rho = \rho_d(T) - \rho_b(T)$ of liquid $^4$He in a wide gap (with a thickness $d_0 = 5.2 \times 10^{-3}$ cm) and in a narrow gap ($d \simeq 3.3 \times 10^{-5}$ cm).

From the theoretical standpoint, there are three effects which would cause the density of helium in a gap to differ from the bulk density: 1) a shift of the temperature of the $\lambda$-transition, $\Delta T_\lambda(d) = T_\lambda - T_\lambda(d)$, due to the boundary condition $\Psi = 0$ on the order parameter (the macroscopic wave function is $\Psi = \eta e^{i\varphi}$) at a solid wall; 2) an inhomogeneity at $T < T_\lambda(d)$ in the spatial distribution of the helium along the $z$ direction, i.e., $\rho_d(z)$, across the gap, reflecting an inhomogeneity of the density of the superfluid part, $\rho_s = m|\Psi(z)|^2$; 3) a change due to the boundary condition $\Psi = 0$ in the spectrum of long-wave fluctuations of $\Psi$, which leads to a "cutoff" of the logarithmic anomaly of the thermal-expansion coefficient $\beta \equiv \rho^{-1}(\partial\rho/\partial T)_p$ for helium in a gap.

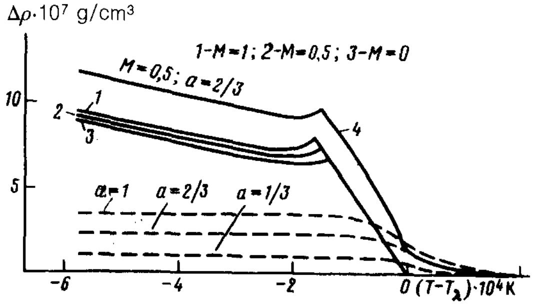

FIG. 1.

The first two of these effects can be calculated quantitatively from the phenomenological $\Psi$-theory of the superfluidity of helium II near the $\lambda$-point.[2] The results of calculations of this sort, which have been carried out for several values of the parameter $M$, which figures in the theory of Ref. 2, are shown by curves 1, 2, and 3 in Fig. 1. Omitting the details of the calculations, which we will report in a more detailed paper, we note that over the interval from $T_\lambda$ to $T_\lambda(d)$ the temperature dependence $\Delta\rho_\Psi(T)$ calculated from the theory of Ref. 2 is described by a straight line $\Delta\rho_\Psi = \rho_\lambda \Delta\beta_b (T_\lambda - T)$ with a slope $\rho_\lambda \Delta\beta_b = 5.3\times 10^{-3}$ g/(cm$^3\cdot$K), where $\Delta\beta_b = \beta_{b,\mathrm{II}} - \beta_{b,\mathrm{I}}$ is the difference between the thermal expansion coefficients of "bulk" HeII and HeI at the same distance from $T_\lambda$. The sharp change in the slope of the $\Delta\rho_\Psi(T)$ curve at $T = T_\lambda(d)$ is due to a discontinuity in the coefficient $\beta_d$ at the $\lambda$-transition of helium in the gap[2]:

$$\Delta\beta_d = \Delta\beta_b \frac{32}{9(3+M)(1-M)} .$$

At $T < T_\lambda(d)$ the difference $\Delta\rho_\Psi(T)$ initially decreases slightly and then increases with distance from $T_\lambda(d)$ in accordance with[2]

$$\Delta\rho_\Psi = Cd^{-1}(T_\lambda - T)^{1/3} ,$$

where the proportionality factor $C$ depends only slightly on the parameter $M$ and has the value $C = 3.08\times 10^{-10}$ g/(cm$^2\cdot$K$^{1/3}$) at $M = 0$.

With regard to the $d$ dependence of the logarithmically divergent part of the thermal expansion coefficient, $\beta_{d,\mathrm{fl}}$, nothing in the way of a detailed quantitative prediction can be made at present. The theory of gauge invariance suggests[3] only that the logarithmic growth of $\beta_{d,\mathrm{fl}}$ will stop in some temperature interval near $T_\lambda$, where the bulk correlation length for the order parameter is $\xi_b(T) \gtrsim d$. Pursuing this suggestion, we find a rough estimate of the fluctuational component of $\Delta\rho(T)$ by assuming that outside the interval $|T_\lambda - T| < a\Delta T_\lambda(d)$ ($a$ is a numerical factor of order unity) the quantity $\beta_{d,\mathrm{fl}}$ is the same as in the bulk material ($\beta_{d,\mathrm{fl}} = \beta_{b,\mathrm{fl}} = A \ln|T_\lambda - T|^{-1}$ with $A = 8.1\times 10^{-3}$ K$^{-1}$; see Ref. 4), while within this interval we have $\beta_{d,\mathrm{fl}} = \mathrm{const} = A \ln[a\Delta T_\lambda(d)]^{-1}$. The temperature dependence of $\Delta\rho_{\mathrm{fl}}(T)$—the component of $\Delta\rho$ due to the long-wave fluctuations of $\Psi$—estimated in this manner is shown by the dashed lines in Fig. 1 for the parameter values $a = 1, 2/3$, and $1/3$. The particular value $a = 2/3$ was chosen to obtain the best fit of the resultant theoretical curve

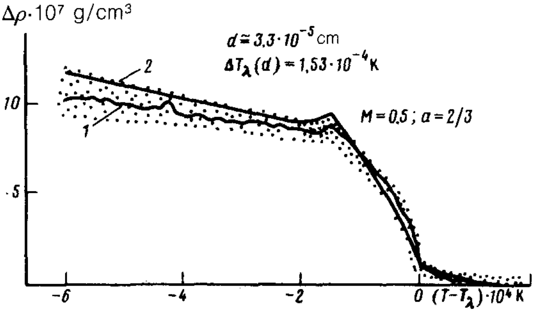

FIG. 2.

$\Delta\rho_{\rm tanh} = \Delta\rho_\Psi + \Delta\rho_{\rm fl}$ (curve 4 in Fig. 1) with the experimental data at $T > T_\lambda(d)$. The component $\Delta\rho_{\rm fl}$ is small in comparison with $\Delta\rho_\Psi$ even at $a = 1$, because the ratio $A/\Delta\beta_b \simeq 0.2$ is small.

For an experimental study of the dependence $\Delta\rho(T)$ we used the same method as was used in Ref. 1 for a quantitative determination of the shift of the $\lambda$-point. A refinement of this method (the details of the present experiments and of the previous experiments will be described in the more detailed paper) made it possible to carry out quantitative measurements of $\Delta\rho$ and to bring out the structural feature in its temperature dependence near the $\lambda$-transition (Fig. 2). In the experiment we measured the difference ($\Delta\epsilon$) in the dielectric constants of helium in wide and narrow gaps, which were parallel-plate capacitors $C_0$ and $C_1$. Any deviations from a parallel arrangement of the plates and from the stated distance between the plates was less than 1.5–2%. The measurements were carried out both at the saturation vapor pressure and at a constant pressure ($p \lesssim 1.5$ atm). The errors in the measurement of the temperature, the pressure, and dielectric constant were $\Delta T \simeq 5\times10^{-7}$ K, $\Delta p \lesssim 1\times10^{-5}$ atm, and $\Delta\epsilon \lesssim 3\times10^{-10}$. The rate of change of the temperature in the measurement of $\Delta\rho$ did not exceed $1\times10^{-3}$ K/min. For the dependence $\Delta\rho(T)$ we observed no hysteresis effects of any sort as the temperature was raised and lowered. For the particular experimental method which we used, the error in the measured difference $\Delta\rho(T)$ comes primarily from the difference between the capacitances of the measurement capacitors. If $C_0 \neq C_1$, a net monotonic error $\Delta\rho_{\rm err} \sim \delta\epsilon(\Delta C/C_0)$ is added to $\Delta\rho(T)$. This error distorts the dependence $\Delta\rho(T)$ [here $\Delta C \equiv C_0 - C_1 \ll C_0$, $\delta\epsilon = \epsilon(T_\lambda) - \epsilon(T)$ is the change in the dielectric constant of $^4$He at a distance $T_\lambda - T$ from the $\lambda$-point]. Under the experimental conditions we have $\Delta C/C_0 \simeq 0.2\%$, so that there would be an error of only 8% in the measured value of $\Delta\rho$ at a distance $\sim 10\Delta T_\lambda(d)$ from the $\lambda$-point.

In the experiments of Ref. 1, whose only purpose was a quantitative determination of the shift $\Delta T_\lambda(d)$, a transitional RC circuit was included in the measurement system to improve the accuracy of the detection of the $\lambda$-transitions. This circuit cut off the constant and slowly varying parts of the signal due to the change in the excess density. As a result, the experimental signal shown by curve 1 in Fig. 2 reflects the qualitative behavior of $\Delta\rho(T)$ in the interval $T_\lambda(d) < T < T_\lambda$, but it leaves undetermined the monotonic change in $\Delta\rho(T)$ outside this interval. In the present experiments, this deficiency was largely eliminated by several refinements of the measurement procedure and through a reduction of the contribution of parasitic effects, one of which was discussed above.

Curve 1 in Fig. 2 shows one of the curves of $\Delta\rho(T)$ near $T_\lambda$ found in the present experiments, after corrections for drift; the dotted band in this figure includes 80% of all the experimental curves found. The value of the parameter $M$ averaged over these curves, with allowance for the effect of the wall potential and electrostrictive effects (the voltage across the capacitor plates was 0.2 V), is $M \simeq 0.5$ . This result agrees with the value $M = 0.6 \pm 0.3$ reported in Ref. 1, estimated there from the magnitude of the shifts of the $\lambda$-point.

The close agreement of the experimental and theoretical curves of $\Delta\rho(T)$, when the long-wave fluctuations are taken into account, is evidence that the theory of Ref. 1 is quite applicable to films and layers of helium. A detailed answer to the question regarding the effect of long-wave fluctuations of $\Psi$ on the behavior of the thermodynamic functions of helium in a bounded geometry will require further research.

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[1] V. I. Panov and A. A. Sobyanin, Pis'ma Zh. Eksp. Teor. Fiz. **35**, 329 (1982) [JETP Lett. **35**, 404 (1982)].
[2] V. L. Ginzburg and A. A. Sobyanin, Usp. Fiz. Nauk **120**, 153 (1976) [Sov. Phys. Usp **19**, 773 (1976)]; J. Low Temp. Phys. **49**, 507 (1982).
[3] M. N. Barber, in Phase Transitions and Critical Phenomena (C. Domb and L. J. Lebowitz, editors), Academic Press, London 1983, Vol. 8, p. 145.
[4] V. I. Panov and A. A. Khvostikov, Zh. Eksp. Teor. Fiz. **83**, 183 (1982) [Sov. Phys. JETP **56**, 99 (1982)].

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