Localized autostructures in homogeneous two-dimensional media

A. V. Gaponov-Grekhov, A. S. Lomov, and M. I. Rabinovich
Institute of Applied Physics, Academy of Sciences of the USSR

(Submitted 17 July 1986)
Pis'ma Zh. Eksp. Teor. Fiz. **44**, No. 5, 242–244 (10 September 1986)

A model describing the creation of spatially localized, stable formations (autostructures) in homogeneous dissipative media is proposed {I. S. Aronson *et al.*, Zh. Eksp. Teor. Fiz. **89**, 92 (1985) [Sov. Phys. JETP **62**, 52 (1985)]; O. V. Vashkevich *et al.*, Dokl Akad. Nauk SSSR (in press)}. The formulated equations are a generalization of the Swift-Hohenberg model [H. S. Greenside and W. M. Coughran, Phys. Rev. A **30**, 398 (1984)] to the two-component media. The physical mechanisms which account for the formation in nonequilibrium media of structures in the form of solitary polyhedra are identified.

1. Can self-nucleation occur in homogeneous two-dimensional media of localized structures (certain formations) which do not depend on the external perturbations? The experiments available (see, e.g., Refs. 2–5) do not give a direct answer to this question: The three-dimensional macro- or microinhomogeneities, localized sources, etc., which are present in each real situation, make it more difficult to unambiguously explain the nature of the observable formations.

The basic possibility of self-nucleation in nonequilibrium two-dimensional media of nontrivial localized structures can be proved by constructing a phenomenological model through which such stable formations can be detected.

We propose here such a model of a homogeneous isotropic medium.

2. Since there are nontrivial autostructures in the form of polyhedra in thermal convection in a nonuniformly heated layer of liquid,[2] we will use the familiar Swift-Hohenberg equation[3,6] as the base of the model which we are seeking:

$$\frac{\partial u}{\partial t} = [\kappa(\mathbf{r}) - (1 + \nabla^2)^2]u + \beta u^2 - u^3 . \tag{1}$$

For $\varkappa = \text{const} > 0$ this equation describes the formation of spatially uniform lattice and the motion of defects in two-dimensional models of a three-dimensional convection.[3,6] As our numerical experiments have shown, this equation, after the addition of a spatially nonuniform increment $\varkappa = -\alpha + v(\mathbf{r})$, can be used as a model for the creation of single elementary autostructures in the form of polyhedra in a layer of fluid with a localized heating. The calculated shape of the cells is the same as that observed in Ref. 2.

To construct a self-consistent model which can describe the formation of localized autostructures in a homogeneous medium (i.e., models without specified inhomogeneities), we should supplement Eq. (1) with an equation for $v(\mathbf{r},t)$ with coordinate-independent parameters which will have spatially localized solutions corresponding to a nonuniform heating if the connection with u is taken into account. As an equation of this sort we can use the equation for a nonlinear homogeneous medium with heat evolution and diffusion[7] and with an additional source $\sim\delta u$,

$$\frac{\partial v}{\partial t} = v - \gamma v^3 + D\Delta v + \delta u. \tag{2}$$

With $\delta = 0$, this equation has localized solutions. These solutions, however, are unstable: Depending on the type of perturbations applied, the excitation either dies out or becomes a homogeneous solution, $v^0 = 1/\sqrt{\gamma}$. Localized solutions of this sort may, nevertheless, turn out to be stable in a self-consistent problem, in which the heat liberated at the periphery of a localized solution $v(\mathbf{r},t)$, for example, is suppressed by the field $u(\mathbf{r},t)$. We thus find the following model-based system:

$$\frac{\partial u}{\partial t} = [(v - \alpha) - (1 + \nabla^2)^2]u + \beta u^2 - u^3, \tag{3}$$

$$\mu \frac{\partial v}{\partial t} = v - \gamma v^3 + \delta u + D\Delta v .$$

To confirm the validity of our ideas about the self-nucleation of localized formations in a homogeneous medium, we first present the results for small β. Autostructures do indeed form here: In the range of parameters considered, they have the shape of a disk, whose scale dimension and steady-state intensity are determined solely by the parameters of the medium and are independent of the boundary and initial conditions (see Fig. 1a).

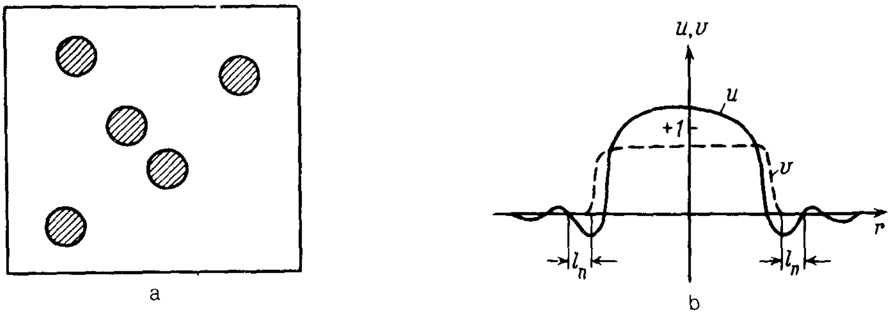

FIG. 1. (a) Autostructures in the form of disks in system (3) with $\beta = 0.9$ (the values of the other parameters are given in Sec. 3); (b) the field distribution $u(\mathbf{r})$ for an autostructure in the form of a disk ($\beta = 0.9$).

The number and relative position of the autostructures, which occur in a reasonably long, two-dimensional medium described by (3), are determined by random initial conditions. These autostructures, however, cannot approach each other closer than the suppression scale l_s, which corresponds to the scale dimension of the region bounding the autostructure, in which the field u is negative (Fig. 1b). The presence of such a region is determined by the particular features of the diffusion u [the terms $(2\Delta u + \Delta^2 u)$ in (2)].[1)] In the suppression region, the nonequilibrium medium does not become self-excited, which accounts for the stability of self-localization of the two-dimensional field u,v.

The nontrivial nature of autostructures is determined by the variety of linear excitations which are the primer for the ensuing nonlinear growth and formation of the autostructure. Polyhedron is the simplest nontrivial structure with a symmetry center. A polyhedron is formed as a result of interaction of only two modes of a round membrane: a radial and an azimuthal mode. In particular, the autostructures in the form of solitary hexahedra and octahedra detected in Ref. 2 can be regarded as the outcome of the interaction of such modes. According to Vashkevich *et al.*,[2] the key factor in the creation of solitary polyhedra is the nonlinearity stemming from the temperature dependence of the surface tension of the fluid. In model (3) such a nonlinearity is taken into account by the term βu^2. Such nonlinearity accounts for the stability of the joint generation of modes which compete with each other at low β.

The arguments advanced above support the fact that self-nucleation and stable existence of localized polyhedra are possible in nonequilibrium homogeneous media described by equations such as (3). Numerical experiments prove this assertion (see Fig. 2).

3. System (2) was solved by a direct difference method with boundary conditions

$$u\big|_S = 0, \qquad \frac{\partial v}{\partial n}\bigg|_S = 0 \tag{4}$$

in a domain with dimensions 40×40. The control counting was performed with the

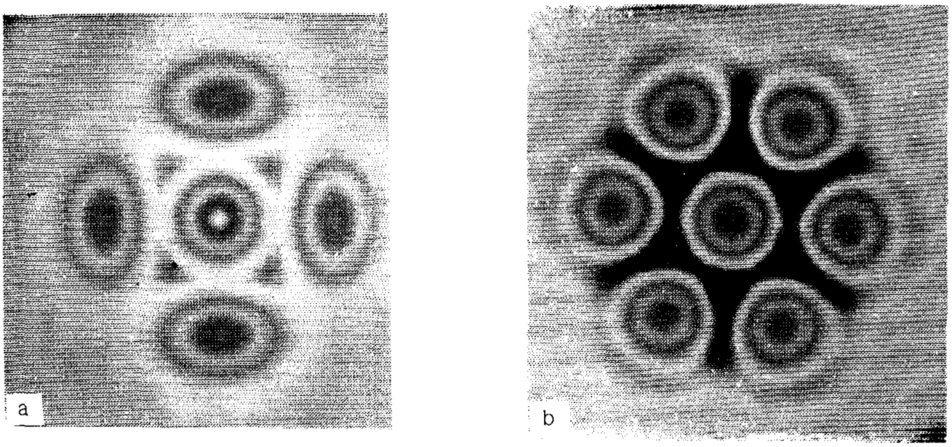

FIG. 2. Autostructures in the form of polyhedra in system (3) with $\beta = 1.5$ under various initial conditions ($\alpha = 0.3, \mu = 0.05, \gamma = 4, \delta = 0.15$, and $D = 0.3$). (a) Tetrahedron—the initial excitation $u(r_0 t_0) \approx 0.75$ is generated inside the circle of radius $R < 7.0$; (b) hexahedron—the initial excitation is generated inside the circle of radius $R > 0.85$.

reduction of the time spacing by an order of magnitude and the coordinate spacing by a factor of two. The counting continued until the system reached a steady-state solution, which usually required up to 80 time units. Typical parameter values are $\alpha \lesssim 0.5$, $\beta = 1.5$, $0.05 \leqslant \mu \leqslant 0.1$, $\gamma = 4$, $\delta = 0.15$, and $D = 0.3$. Under random initial conditions the system produced polyhedra with different number of sides, randomly dispersed along the entire field.

The initial conditions were chosen carefully in order to identify the domains of "attraction" of the various structures. In particular, we found that if the parameters are the same, the system will form either one of the elementary autostructures in the form of a tetrahedron or hexahedron (Fig. 2) or their compositions in the form of a pentahedron, depending on the initial field distribution in the medium. A polyhedron has an arbitrary orientation in space and its parameters remain constant when the boundary conditions and the domain size are changed.

Autonomy of a two-dimensional field in system (3) thus leads to the formation in a homogeneous medium of autostructures in the form of solitary polyhedra. The physical mechanisms which underlie this autonomy and which are constrained by the nonlinearity and the particular features of the diffusion are sufficiently general mechanisms which can operate in various two-component media.

[1)]Superposition of the terms $\nabla^2 u$ and $\nabla^4 u$ actually means that the medium has an independent spatial scale, i.e., a spatial dispersion.

[1]I. S. Aranson, A. V. Gaponov-Grekhov, and M. I. Rabinovich, Zh. Eksp. Teor. Fiz. **89**, 92 (1985) [Sov. Phys. JETP **62**, 52 (1985)].
[2]O. V. Vashkevich, A. V. Gaponov-Grekhov, A. B. Ezerskiĭ, and I. M. Rabinovich, Dok. Akad. Nauk SSSR, 1986 (in press).

[3]H. S. Greenside and W. M. Coughran, Phys. Rev. A **30**, 398 (1984).
[4]F. V. Bunkin, N. A. Kirichenko, and B. S. Luk'yanchuk, Usp. Fiz. Nauk **138**, 45 (1982) [Sov. Phys. Usp. **25**, 662 (1982)].
[5]A. Pocheau, V. Croquette, and P. Le Gal, Phys. Rev. Lett. **55**, 1094 (1985).
[6]H. Haken, Phys. Scripta **T9**, 111 (1985).
[7]S. P. Kurdyumov, in Sovremennye problemy matematicheskoĭ fiziki i vychislitel'noi matematiki (Modern Problems in Mathematical Physics and Computational Mathematics), Nauka, Moscow, 1982.

Translated by S. J. Amoretty