Magnetic resonance and spin waves in thin layers of superfluid ^3He-B

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A shift of the frequency of a transverse magnetic resonance arises in a thin layer (film) of superfluid ^3He-B. This shift depends on the thickness of the layer and is a consequence of a surface dipole interaction. The spectra of spin waves in thin layers of ^3He-B have been studied. A possible experimental determination of the constants of the surface dipole interaction is discussed.

The dipole interaction of nuclear spins leads to a frequency shift of a uniform magnetic resonance of superfluid phases of ^3He. Both a longitudinal resonance and a frequency shift of the transverse resonance (a shift from the Larmor frequency) result from this interaction in the anisotropic A-phase. In the isotropic B-phase, for small spin oscillations, the dipole interaction determines the frequency of the longitudinal resonance, Ω_D, but is does not shift the frequency of the transverse resonance.[1] Solid walls (or a free surface) bonding superfluid ^3He have an orienting effect on the distribution of the order parameter in their vicinity. In the case of the B-phase, the orienting effects gives rise to an induced anisotropy: The direction normal to the wall becomes a special direction. The interaction of the order parameter with the wall can be described by introducing an additional surface dipole energy,[2,3] which should evidently also affect an NMR in such a system. In the present letter we calculate the spin-wave spectra in a ^3He-B layer or arbitrary thickness (with respect to all the characteristic lengths), taking the volume and surface dipole energy into account. The results show, in particular, that a gap appears in the spectra of small transverse spin oscillations in sufficiently thin layers of ^3He-B. This gap depends on the thickness of the layer and is a consequence of a surface dipole interaction. The presence of this gap in spin-wave spectra (a shift of the frequency of the uniform transverse resonance in an external magnetic field directed normal to the layer) is a consequence of an effective anisotropy of ^3He-B which arises in thin layer (or films). This phenomenon is intimately related to the possible existence of a special anisotropic ordering, similar to the ordering in the A-phase and surface layer of ^3He-B (Refs. 4 and 5).

We consider a layer of ^3He-B of thickness h, bounded by two identical solid walls. In the absence of an external magnetic field (or if the field is directed along the normal to the plane of the layer), the equilibrium texture will be a uniform texture in which the vector **n**, along the axis around which the spin space is rotated [through an angle arc cos($-$ 1/4)] with respect to the orbital space, is directed along the normal to the boundary (the Z axis). In the case of a uniform equilibrium distribution of the vector **n**, the linear spin dynamics of the B-phase can be described conveniently by the three-dimensional vector φ_i of small rotation angles: The components φ_x and φ_y describe the direction of the unit vector **n**, while the component φ_z describes the rotation around this vector.[6] In terms of these variables, the Lagrangian of the system in the harmonic approximation, in which the volume and surface dipole energies are taken into account, can be written[1,7,8]

$$L = \int \frac{\chi}{2\gamma^2}\left\{\left(\frac{\partial\varphi_i}{\partial t}\right)^2 - c_t^2 \frac{\partial\varphi_i}{\partial x_k}\frac{\partial\varphi_i}{\partial x_k} - \frac{1}{2}(c_e^2 - c_t^2)\left(\frac{\partial\varphi_i}{\partial x_k}\frac{\partial\varphi_k}{\partial x_i} + \frac{\partial\varphi_i}{\partial x_i}\frac{\partial\varphi_k}{\partial x_k}\right) - \Omega_D^2\varphi_z^2\right\} dV - \int \frac{\chi}{2\gamma^2}\Omega_s^2(\varphi_x^2 + \varphi_y^2)dS, \quad (1)$$

where χ is the susceptibility of the ^{3}He-B, γ is the nuclear gyromagnetic ratio, c_e and $c_t = \sqrt{2}c_e$ are the velocities of longitudinal and transverse spin waves, and the constant Ω_s^2 describes (in this approximation) the surface dipole interaction. By varying (1), we find the equations of volume spin oscillations which incorporate the dipole interaction, and we also find the boundary conditions expressing the conservation of the spin current across the boundaries, where the surface dipole interaction is taken into account. The general solution for a spin wave in the layer can be found as a superposition of the solutions of volume equations of the form $\varphi_i \propto \exp(ikx - i\omega t \pm i\kappa_j z)$, where ω and κ, are the frequency and wave number of the spin wave, and κ_j ($j = 1,2,3$) are the eigenvalues of the characteristic equation of the volume oscillations. Taking the equivalence of the boundaries of the layer and the form of the volume equations for spin oscillations into account, we can break up the general solution into three waves. One of them is polarized (for the vector $\vec{\varphi}$) in the plane of the layer perpendicular to the wave vector. The corresponding dispersion relation is

$$\tan\kappa_3 h = 2\Omega_s^2 c_t^2 \kappa_3 / (c_t^4\kappa_3^2 - \Omega_s^4), \qquad \kappa_3 = (\omega^2/c_t^2 - k^2)^{1/2}.$$

The two other waves ("symmetric and antisymmetric") are polarized in the orthogonal plane. Their dispersion relations are more complex, and we will not reproduce them here. The direction of the magnetization oscillations in a spin wave can be found from the direction of the oscillations of the orbital-space vector $\vec{\varphi}$ through a rotation by an angle arc cos($-$ 1/4) around **n**. The distribution (in the bulk of the layer) of the small-rotation vector $\vec{\varphi}$ in these spin waves is analogous to the distribution of the displacement vector **u** in Lamb waves in an elastic layer.

The gap which arises in the spectra of transverse spin oscillations as a result of the surface dipole interaction arises in layers of thickness $h \ll h_0$, where $h_0 \sim c_t^2/\Omega_s^2$. Since $\Omega_s^2 \sim \Omega_D^2\xi_0$ (Refs. 2 and 3), the characteristic thickness is $h_0 \sim \xi_D^2/\xi_0 \gg \xi_D$ (ξ_0, ξ_D) are the correlation and dipole lengths). In the case $kh \ll 1$, $h \ll h_0$ for example, the spectra $\omega(k)$ of the lowest-lying longitudinal spin wave and of the two transverse spin waves in the layer are

$$\omega_e^2 = \Omega_D^2 + \frac{15}{8}c_e^2 k^2, \quad (2)$$

$$\omega_{t1}^2 = \frac{2\Omega_s^2}{h} + \frac{3}{8}c_t^2 k^2, \qquad \omega_{t2}^2 = \frac{2\Omega_s^2}{h} + c_t^2 k^2.$$

The absence of a branch with a quadratic dispersion[8] from the spectra of transverse spin waves in thin layers of ^{3}He-B at $kh \ll 1$ is a direct result of the asymmetry of the spin current-density tensor (in contrast with the symmetry of the elastic stress tensor in a solid). It can be seen from (2) that the gap in the spectra of transverse spin oscillations, $\omega_{t1,2}(0) = \Omega_s(2/h)^{1/2}$ is small in the case $h \gg \xi_0$ [at $h \sim 1$ μm, for example, we would have $\omega_t(0) \sim 0.1\Omega_D$], but it increases with decreasing layer thickness, becoming comparable to Ω_D at $h \sim \xi_0$; i.e., the superfluid ^{3}He-B in a layer (or film) of such a small thickness is indistinguishable from the A-phase in an NMR experiment. If an external magnetic field **H** is directed along the normal to the plane of the layer, the frequencies of the uniform transverse resonance split, becoming

$$\omega_{t1,2}(0) = \pm \frac{1}{2}\gamma H + \left[\left(\frac{1}{2}\gamma H\right)^2 + \frac{2\Omega_s^2}{h}\right]^{1/2}.$$

Measurements of the frequency shift of the transverse resonance in thin layers of ^{3}He-B and of its dependence on the thickness would thus make it possible to experimentally determine the magnitude of the surface-dipole-interaction constant. The induction signal from a thin layer of helium could be intensified by using a sandwich structure (consisting of a layer of helium, a solid layer, and another layer of helium, etc).[1)]

In thicker layers of ^{3}He-B ($h \gg h_0$), the gaps in the spin-wave spectra (at $kh \ll 1$) are determined not by a dipole interaction but by size quantization: $\omega_{e,t}(0) \sim \pi c_{e,t}/h$. Spin waves in a layer of arbitrary thickness have also been studied in the short-wave limit, $kh \gg 1$, which corresponds to surface spin waves. In a layer of arbitrary (but finite) thickness, there is no point at which the spin-wave spectrum ends[8]: There is a real solution for the spin-wave spectrum ends[8]: There is a real solution for the spin-wave frequency at an arbitrary value of k. At wavelengths $k^{-1} \ll (h\xi_D)^{1/2}$, $\xi_D \ll k^{-1} \ll \xi_D(\xi_D/\xi_0)^{1/2}$, surface spin waves have some distinctive features: They penetrate deeply (the reciprocal of the penetration depth is $\kappa_e \sim k^2\xi_D \ll k$), and their polarization and velocity are very nearly the same as those of a bulk longitudinal spin wave ($\omega = c_e k$) that propagates in the same direction. In this wavelength interval, these distinctive features are due entirely to the pronounced anisotropy of the phase velocities of bulk spin waves, which is a consequence of the dipole interaction at $k\xi_D \ll 1$. This circumstance is independent confirmation of the condition which has been established for the existence of deeply penetrating surface elastic waves in highly anisotropic crystals.[9] This circumstance means that we can extend this condition to surface waves which are not acoustic waves.

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