orrelation functions in the one-dimensional ubbard model

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The asymptotic forms of the correlation functions in the one-dimensional Hubbard model, when umklapp processes are significant, are obtained. It is shown that two types of pairing in the ground state are simultaneously realized. The corresponding correlation functions fall off in power-law fashion at large distances.

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The asymptotic forms of the correlation functions were recently calculated[1] for a one-dimensional electron gas in the absence of umklapp processes (see also[2,3]). The kinetic energy of the free electrons was linearized near $\pm k_F$, and this made it possible to use the so-called "boson representation" for the field operators[4,5]

$$\psi_{is} = L^{-1/2}\Sigma\, e^{ikx} a_{is}(k) \to (2\pi a)^{-1/2}\exp\{(-1)^i[-ik_F x + \Sigma A(x,k)\rho_{is}(k)]\}. \quad (1)$$

The exponent $i=(1,2)$ corresponds here to electrons near $\pm k_F$, s is the spin index, $\rho_{is}(k)=\Sigma_p a^+_{is}(p+k)a_{is}(p)$, $A(x,k)=2\pi L^{-1}k^{-1}\exp(-\alpha\,|k|/2-ikx)$, and $v_F\alpha^{-1}$ is interpreted as the width of the conduction band.

It will be shown below that the boson representation technique makes it possible to find the asymptotic forms of the correlation functions also in those cases when the umklapp processes are significant. For simplicity we consider the one-dimensional Hubbard model with a half-filled band. The interaction constant of the electrons will be assumed to be small: $|g|\ll 2\pi v_F$. Such a system corresponds in the representation (1) to a Hamiltonian[6]

$$H = h\{\rho_i; -g\} + h\{\sigma_i; g\}, \quad (2)$$

where

$$h\{\rho_i; -g\} = 2\pi(v_F + g/2\pi)L^{-1}\sum_{k>0}[\rho_1(k)\rho_1(-k) + \rho_2(-k)\rho_2(k)]$$

$$+ gL^{-1}\Sigma\rho_1(k)\rho_2(-k) - g(2\pi a)^{-2}\int dx\{\exp[2^{1/2}\Sigma A(x,k)(\rho_1(k)+\rho_2(k))] + \text{c.c.}\}, \quad (3)$$

$h\{\sigma_i;g\}$ is determined by replacing ρ_i in (3) by σ_i and g by $-g$; $\rho_i = 2^{-1/2}(\rho_{i\uparrow}+\rho_{i\downarrow})$, $\sigma_i = 2^{-1/2}(\rho_{i\uparrow}-\rho_{i\downarrow})$, where

$$[\rho_i, \sigma_j] = 0, \quad (4)$$

$$[\rho_i(k), \rho_j(-k')] = [\sigma_i(k), \sigma_j(-k')] = (-1)^i\delta_{ij}\frac{kL}{2\pi}\delta_{kk'}.$$

We note that in the Hubbard Hamiltonian (2) the umklapp processes correspond to the last term in $h\{\rho_i; -g\}$.

We are interested in the correlation functions that describe the fluctuations of singlet and triplet Cooper pairs (SCP and TCP), as well as fluctuations of the dielectric (CDW) and antiferromagnetic (SDW) type:

$$K_{CDW} = < \psi_{1\uparrow}(x,t)\,\psi^+_{2\uparrow}(x,t)\,\psi_{2\uparrow}(0.0)\,\psi^+_{1\uparrow}(0.0) > = e^{2ik_F x} K^+_\rho(-g)\,K^+_\sigma(g),$$

$$K_{SDW} = < \psi_{1\uparrow}(x,t)\,\psi^+_{2\downarrow}(x,t)\,\psi_{2\downarrow}(0.0)\,\psi^+_{1\uparrow}(0.0) > = e^{2ik_F x} K^+_\rho(-g)\,K^-_\sigma(g),$$

$$K_{SCP} = < \psi_{1\uparrow}(x,t)\,\psi_{2\downarrow}(x,t)\,\psi^+_{2\downarrow}(0.0)\,\psi^+_{1\uparrow}(0.0) > = K^-_\rho(-g)\,K^+_\sigma(g), \tag{5}$$

$$K_{TCP} = <\psi_{1\uparrow}(x,t)\,\psi_{2\uparrow}(x,t)\,\psi^+_{2\uparrow}(0.0)\,\psi^+_{1\uparrow}(0.0) > = K^-_\rho(-g)\,K^-_\sigma(g),$$

where

$$K^\pm_\rho(-g) = (2\pi a)^{-1} < e^{ith}\exp[-2^{-1/2}\Sigma A(x,k)(\rho_1 \pm \rho_2)]\,e^{-ith} \times \exp[2^{-1/2}\Sigma A(0,k)(\rho_1 \pm \rho_2)] >_n, \tag{6}$$

here $h = h\{\rho_i; -g\}$. $K^\pm_\sigma(g)$ is obtained from (6) by replacing ρ_i with σ_i and $h = h\{\sigma_i; g\}$.

The factorization in (5) was made possible by the fact that the field operators ψ_{is} are expressed in accordance with (1) in terms of an exponential of a function linear in ρ_{is}. Since the commutation relations of ρ_i and σ_i are the same, $K^\pm_\rho(-g) = K^\pm_\sigma(-g)$ and consequently the problem reduces to finding the four functions $K^\pm_\sigma(g \gtrless 0)$.

The $K^\pm_\sigma(g>0)$ can be calculated, inasmuch as a situation of the zero-charge type arises at $g>0$ following normalization in $h\{\sigma_i; g\}$. For the calculations we note that the parquet approximation was sufficient to find the correlation function in a gas with δ-function repulsion[7-9]

$$\widetilde{K}_{CDW}(g>0) \sim (\ln\omega)^{-3/2}\,\omega^{-g/2\pi v_F}, \qquad \widetilde{K}_{SDW}(g>0) \sim (\ln\omega)^{1/2}\,\omega^{-g/2\pi v_F} \tag{7}$$

On the other hand, the factorization relations of the type (5) are valid also for these functions, the factors $K^\pm_\sigma(g)$ remaining unchanged, while $K^\pm_\rho(-g)$ are replaced by $\widetilde{K}^\pm_\rho(-g)$. The $\widetilde{K}^\pm_\rho(-g)$ are calculated from formula (6) with $h = \widetilde{h}\{\rho_i - g\}$; while $\widetilde{h}\{\rho_i; -g\}$ are determined by formula (3) but without the last term (this term describes umklapp processes, which do not exist in a gas with δ-function interaction). $\widetilde{h}\{\rho_i; -g\}$ is quadratic in ρ_i, and therefore the function $\widetilde{K}^+_\rho(-g)$ can be easily obtained[4]

$$\widetilde{K}^+_\rho(-g) \sim \omega^{-1,\,-g/2\pi v_F}\,. \tag{8}$$

As a result we obtain from (7) and (8) the asymptotic forms of $K^\pm_\sigma(g>0)$

$$K^+_\sigma(g>0) = \widetilde{K}_{CDW}(g>0)/\widetilde{K}^+_\rho(-g) \sim \ln^{-3/2}[x^2-(v't)^2]/[x^2-(v't)^2]^{1/2}, \tag{9}$$

$$K^-_\sigma(g>0) = \widetilde{K}_{CDW}(g>0)/\widetilde{K}^+_\rho(-g) \sim \ln^{1/2}[x^2-(v't)^2]/[x^2-(v't)^2]^{1/2}, \tag{10}$$

where $v' = v_F - g/2\pi$.

Reducing the problem to a two-dimensional Coulomb gas, it was shown in[3] that at large distances we have the asymptotic behavior

$$K^{+}_{\sigma}(g < 0) \sim \text{const} \ . \tag{11}$$

It remains to explain the behavior of the function $K_{\sigma}^{-}(g<0)$. It was noted in[2] that at $g=-(6/5)\pi v_F$ it is possible to calculate $K_{\sigma}^{-}(x,t)$. This circumstance makes it possible to find the asymptotic form of $K_{\sigma}^{-}(g<0)$, inasmuch as at $g<0$, as the result of renormalization, the charge arrived at the point $\tilde{g}=-\frac{6}{5}\pi \tilde{v}_F$, ($\tilde{v}_F$ is the coefficient at

$$2\pi L^{-1} \sum_{k>0} [\sigma_1(k)\sigma_1(-k)+\sigma_2(-k)\sigma_2(k)]$$

in the renormalized $h\{\sigma_i;\tilde{g}\}$). Omitting certain details (we note only that $\tilde{v}_F = 1.25(v_F - g/2\pi) + O(g^2)$), we present the final result

$$K_{\sigma}^{-}(g < 0) \sim \frac{1}{(x/v'')^2 - t^2} \exp\{-2\Delta[(x/v'')^2 - t^2]^{1/2}\}, \tag{12}$$

where

$v'' = v_F + |g|/2\pi$ and $\Delta \sim a^{-1}|g\, v_F|^{1/2}\exp(-\pi v_F/|g|)$

is the gap in the spectrum of the fermion excitations $\epsilon(p) = [\Delta^2 + (v''p)^2]^{1/2}$.

It follows from (5) that $K_{\sigma}^{-}(g<0)$ enters in $K_{\rm SCP}(g>0)$, $K_{\rm SDW}(g<0)$, and $K_{\rm TCP}(g \lessgtr 0)$. These functions fall off exponentially at large distances; in addition

$$\mathrm{Im} K^r_{\rm SCP}(g > 0), \quad \mathrm{Im} K^r_{\rm SDW}(g < 0), \quad \mathrm{Im} K^r_{\rm TCP}(g \gtrless 0) \sim \theta(|\omega| - 2\Delta), \tag{13}$$

K^r is the susceptibility and describes the response of the system to the action of the corresponding external field.

In the remaining cases we obtain a power-law decrease of the correlation functions

$$K_{\rm CDW}(g \gtrless 0) \sim e^{2ik_F x} \ln^{-3/2}[x^2 - (v't)^2]^{-}/[x^2 - (v't)^2]^{1/2}, \tag{14}$$

$$K_{\rm SDW}(g > 0) \sim e^{2ik_F x} \ln^{1/2}[x^2 - (v't)^2]/[x^2 - (v't)^2]^{1/2}, \tag{15}$$

$$K_{\rm SCP}(g < 0) \sim \ln^{1/2}[x^2 - (v't)^2]/[x^2 - v't)^2]^{1/2}, \tag{16}$$

where $v'_F = v_F - |g|/2\pi$ is the velocity of the gapless excitations.

The slow (power-law) decrease of the correlation function at large distances means that a corresponding type of pairing is realized in the system, although no long range order is produced, owing to the strong quantum fluctuations inherent in a one-dimensional system. It follows from (14)—(16) that two types of pairing in the Hubbard model are realized simultaneously: SCP and CDW in the case of attraction, and SDW and CDW in the case of repulsion. To excite waves of another type it is necessary, according to (13), to expend an energy 2Δ to break the pair.

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