Phase transitions in quasi-one-dimensional system of weakly bound one-dimensional metallic filaments

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We investigate the dependence of the phase-transition temperature T_c in a quasi-one-dimensional system on the value w of the transverse kinetic coupling, which characterizes the ease of tunneling of electrons from filament to filament. With allowance for one-dimensional fluctuation effects (carried out in second order of the renormalization-group method), it is shown that T_c can have a maximum in the region of small w.

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It is known that in one-dimensional systems and in a system of weakly bound one-dimensional filaments, it is necessary to take simultaneously into account the Cooper and the Peierls instabilities,[1,2] the fluctuation effects, and the special role of collective excitations.[3] We investigate below, by the method proposed in[4], the dependence of T_c on w with allowance for all these factors, in a wide range of values of w.

Let the one-dimensional filaments be packed into a flat lattice with period a. Neglecting tunneling between the filaments, the electron spectrum is planar. Allowance for the transition between filaments leads to a corrugation of the Fermi surface

$$\epsilon(p) - \epsilon_F = v_F(|p_\parallel| - p_F) + w(p_\perp). \quad (1$$

Here $p_\parallel$ is the projection of the electron momentum along the filament; v_F and p_F are the Fermi velocity and momentum in this direction, and depend little on $p_\perp$ if $w/\epsilon_F \ll 1$. Two qualitatively different cases are possible. In case A, the degenerate form of the electron spectrum, typical of a one-dimensional system, is clearly preserved:

$$\epsilon(p) - \epsilon_F = -[\epsilon(p - q_0) - \epsilon_F].$$

This causes anomalous scattering in the electron-hole channel, characterized by a momentum q_0, and leads to the appearance of logarithmic singularities. For example, if we put

$$w(p_\perp) = -2w\left[\cos\left(\frac{a p_x}{\hbar}\right) + \cos\left(\frac{a p_y}{\hbar}\right)\right]$$

(here $q_0 = 2p_F$, $\pi\hbar/a$, $\pi\hbar/a$), then the corresponding singularities correspond to anti-Peierls instability. We shall designate the case in which (2) is not satisfied by B.

An important role is played here by the interaction of electrons belonging to different Fermi surfaces, the total vertex part of the interaction being of the form

$$\Gamma(b+c \to b+c) = -\frac{1}{2}\left[\gamma_1 g_1 \sigma_b^z \sigma_c^z + \gamma_2 g_2 I_b I_c\right] - \gamma_4 g_4 [\sigma_b^+ \sigma_c^- + \sigma_b^- \sigma_c^+], \quad (3)$$

where the letters b and c denote the electrons situated at the Fermi surfaces $\pm p_F$; σ^z and $\sigma^\pm = \sigma^x \pm i\sigma^y$ are Pauli matrices, and I is a unit matrix, all acting in the space of the spins of the electrons b and c. The amplitudes $\gamma_1 g_1$ and $\gamma_2 g_2$ describe interaction processes with small momentum transfer, $\gamma_1 g_1$ corresponding to that part of the scattering which is antisymmetrical in the spins, and $\gamma_2 g_2$ to the symmetrical part; $\gamma_4 g_4$ corresponds to processes with transfer of a large longitudinal momentum $\approx 2p_F$. In (3) we separated the vertex function γ_i and the "charge" g_i. The model investigated in[5] corresponds to the following choice of charges $\{g_i'\}$: $g_1' = g_1 = g_4$; $g_2 = g_1' - 2g_2'$. The bare coupling constants g_i^0, which are assumed to be small, are determined at a "cutoff" energy ω_D.

The investigation of the thermodynamic instabilities of the system reduces to a study of the vertex functions, the approximately correct behavior of which is obtained in the one-dimensional case with the aid of the renormalization-group method.[5,6] The invariant "charges" g_i satisfy the Lie differential equations, and the function $\Psi\{g_i\}$ of Gell-Mann and Low can be approximated in the quasi-one-dimensional case in the following manner.[1)] At $T \gg w$ it coincides with the corresponding expression obtained for $\Psi\{g_i\}$ for a one-dimensional system in third order in g_i.[5] In the region $T \ll w$ we confine ourselves to the second order in the expansion of $\Psi\{g_i\}$ in powers of g_i. This is equivalent, in the sense of the phenomenological theory of phase transitions, to the molecular-field approximation. In this region, the vertex functions and the invariant "charges" behave in the same manner. Thus, we have the following equations (case B)[2)]: at $x \equiv T/\omega_D > y \equiv w/\omega_D$ we have

$$x\frac{d}{dx}g_1 = 2g_4^2(1 + g_1); \quad x\frac{d}{dx}g_4 = g_1 g_4(2 + g_1) + g_4^3; \quad x\frac{d}{dx}g_2 = 0 \quad (4)$$

and at $x > y$

$$x\frac{d}{dx}g_1 = g_4^2 - g_1 g_2; \quad x\frac{d}{dx}g_2 = -\frac{1}{2}(g_1^2 + g_2^2) - g_4^2; \quad x\frac{d}{dx}g_4 = g_1 g_4 - g_2 g_4. \tag{5}$$

The initial conditions for (4) are $g_i(x=1) = g_i^0$. The quantity g_i depends on y only if $x < y$, since the solutions of the systems (4) and (5) must be matched at $x = y$.

The system can be easily integrated; we obtain[3)]

$$g_1(x, y) + g_2(x, y) = \frac{g_1(y) + g_2(y)}{1 + \frac{1}{2}(g_1(y) + g_2(y))\ln\frac{x}{y}}, \tag{6}$$

$$g_1(x, y) - g_2(x, y) \pm 2g_4(x, y) = \frac{g_1(y) - g_2(y) \pm 2g_4(y)}{1 - \frac{1}{2}(g_1(y) - g_2(y) \pm 2g_4(y))\ln\frac{x}{y}}. \tag{7}$$

The three pole singularities correspond to the three possible realizations of ODLRO in the system: pairing of the electrons in the triplet state with total-spin projection equal to ± 1 and 0, and in the singlet state. The corresponding critical temperatures, apart from the pre-exponential factor, are equal to

$$T_c^{S=1;\ S_z=1} = w\exp\left[-\frac{2}{g_1(y) + g_2(y)}\right];$$

$$T_c^{S=0,1;\ S_z=0} = w\exp\left[-\frac{2}{g_2(y) \pm 2g_4(y) - g_1(y)}\right]. \tag{8}$$

The solution of the system (4) cannot be written down in explicit form, and we indicate only the following properties of these solutions: $g_1(x)$, $g_4(x) \to -1$ at $g_1^0 < |g_4^0|$ in the limit as $x \to 0$ and $g_4(x) \to 0$, and $g_1(x) \to (C/2) + (C^2/4 + C)^{1/2}$ at $g_1^0 \geq |g_4^0|$ in the limit as $x \to 0$. Here $C = ((g_1^0)^2 - (g_4^0)^2)(1 + g_1^0)^{-1} = (g_1^2(x) - g_4^2(x)) \times (1 + g_1(x))^{-1}$ is the first integral of motion of the system (4).

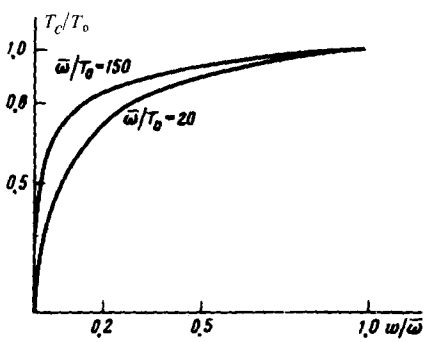

FIG. 1. Dependence of T_c on w in the case A for the Little mechanism of interaction.

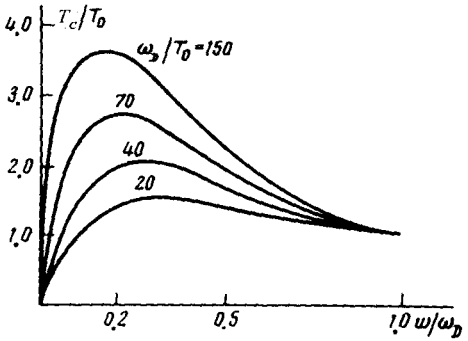

FIG. 2. Dependence of T_c on w in case B for the phonon interaction mechanism.

In case A, at $x>y$, (4) remains in force; in the region $x<y$, with allowance for the anti-Peierls instability, we have

$$x\frac{d}{dx}g_1 = 2g_4^2; \qquad x\frac{d}{dx}g_2 = 0; \qquad x\frac{d}{dx}g_4 = 2g_1g_4 \quad . \tag{9}$$

The pole singularity in the solution arises at $g_1^0 < |g_4^0|$ and corresponds to simultaneous instability relative to the anti-Peierls doubling and the Cooper pairing. For T_c we have

$$T_c(w) = w\exp\{(g_4^2(y) - g_1^2(y))^{-1/2}\operatorname{arc\,cth}(2g_1(y)[g_4^2(y) - g_1^2(y)]^{-1/2})\}. \tag{10}$$

Thus, the transition temperature depends on the three-dimensionality parameter and on the magnitude and character of the interaction. We shall analyze this dependence for two interaction mechanisms: the phonon mechanism and the "Little" mechanism. If the interaction is due to exchange of phonons with large ($\approx 2p_F$) longitudinal momentum transfer, then $g_1^0=g_2^0=g_4^0=-\nu$ and are limited to frequencies on the order of the Debye frequency ω_D. The Little mechanism presupposes that $g_1^0=-g_2^0=g_4^0=-\lambda$, [1-5] and the cutoff energy $\bar{\omega}$ is of the order of ϵ_F.

For the Little mechanism, T_c increases in the interval $0<w<\bar{\omega}$ monotonically from 0 to $T_0=\bar{\omega}\exp(-\frac{1}{2}\lambda)$, which is the value of the critical temperature obtained in the parquet approximation for a one-dimensional system. [1] It is shown in Fig. 1 for case A and is described by the equation [here $\Delta=T_0(\lambda/(1-\lambda))^{1/2}$]:

$$1+\left(\frac{T_c}{\Delta}\right)^2+\ln\left(\frac{T_c}{w}\right)^2=0. \tag{11}$$

For the phonon mechanism in case A, the function $T_c(w)$ is described by (11) with the substitution $\lambda\to\nu$. In the case B, the function $T_c(w)$ is shown in Fig. 2. It has a maximum at $w=w_m<\omega_D$, with $T_m\equiv T_c(w_m)>T_0\equiv T_c(\omega_D)$. Using (4) and (8), we can obtain for w_m and T_m the following system of equations:

$$T_m = w_m \exp\left(\frac{2}{\nu - 3\nu_m}\right), \tag{12}$$

$$(3\nu_m - \nu)^2 = 12\nu_m^2(1 - \nu_m); \qquad w_m = T_0\left(\frac{\nu}{\nu_m}\,\frac{1-\nu_m}{1-\nu}\right)^{1/2} \exp\left(\frac{1}{2\nu_m} + \frac{1}{2\nu}\right),$$

where $T_0 = \omega_D \exp(-1/\nu)$ corresponds to the transition temperature for a strongly anisotropic superconductor, and we choose from among the solutions of (12) the one for which $\nu_m > \nu$.

The increase of T_c is due to the fact that in this type of system there is an additional renormalization of the coupling constants, due to the presence of singularities of the Peierls type in the temperature region $w < T < \omega_D$,[4)] which effectively increases the interaction constants, and by the same token also T_c. At $w < w_m$, in the temperature region $w \leq T \leq w_m$, an important role is played also by longitudinal density fluctuations, which limit the subsequent increase of the coupling constants, and it is this which leads to a decrease of T_c at $w < w_m$.

The present approximation is confined to values $w > \Delta$ (see (11)). At $w \leq \Delta$ all the g_i become ≈ 1, and Eqs. (4), (5), and (9) no longer suffice. We note that the maximum on Fig. 2 is located in the region where the theory is applicable. It is possible that a reasonable approximation at $w \ll \Delta$ is provided by the method of the average field with respect to w, in the spirit of[3,8]

[1)]A rigorous proof calls for introduction of scaling with respect to the parameter.[7]

[2)]We have introduced here the dimensionless "charges" by multiplying the initial g_i by $\rho/4$, where $\rho = 2/\pi\hbar v_F a^2$ is the density of states on the Fermi surface.

[3)]The fact that Eqs. (6)—(8) and (10) contain $g_i(y)$ and not g_i^0 seems to correspond to the moving-pole idea.[2] Formulas (4) give a concrete prescription for finding the position of the moving pole.

[4)]Renormalizations of this type, which are responsible, for example, for the softening of the phonon mode, arise in the region $T > \omega_D$. It is assumed here that they are included in ν. Such renormalizations can be consistently taken into account within the framework of the two-limit technique (see, e.g., [6]).

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