Superfluidity of Bose condensate of electron-hole pairs

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It is shown that the rather weak processes that fix the phase of the wave function of an electron-hole pair (interband transitions in an excitonic dielectric) admit of the existence of metastable spatially inhomogeneous states with nonzero total electron quasimomentum.

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Superfluidity of electron-hole pairs was discussed in the literature for three cases: 1) high-density excitons obtained under nonequilibrium conditions (see the reviews[1,2]); 2) the excitonic dielectric produced from a metal or semiconductors when the width of the forbidden band is less than the binding energy of the exciton (see the review[3]); 3) spatially-separated electrons and holes.[4,5] The interband transitions, as shown by Guseĭnov and Keldysh[6] lift the phase degeneracy of the order parameter in the excitonic dielectric, i.e., they "fix" the phase, making impossible the existence of spatially homogeneous states with constant phase gradients, such as current states of an ordinary superfluid. For the third case, a similar role is played by tunnel transitions.[1)] However, as we shall show below, so long as the processes that fix the phase remain quite weak, spatially inhomogeneous states analogous to current superfluid states can nevertheless exist in the sense that they differ in energy from the ground state, are metastable, and are characterized by a volume-averaged value of a certain vector that is connected with the complex order parameter $\Psi=\sqrt{n_s}\exp(i\phi)$ by the same relation as the superfluid mass flux in an ordinary superfluid, whereas the superfluid mass flux equals the momentum density, the aforementioned vector equals the quasimomentum electron density. For an almost empty band, the quasimomentum has the same direction as the electron mass flux, and for an almost filled band the two directions are opposite. In the case of an excitonic dielectric, the quasimomentum density corresponds therefore to the exciton flux. This flux influences the dielectric gap and can be produced in experiment, even though it carries neither a real mass, nor charge, nor, in the case of total equilibrium with respect to the interband transitions, energy.

We confine ourselves below to the case when the order parameter can be etermined from the Ginzburg-Landau equation, i.e., when it corresponds to he minimum of the functional

$$E = \int d\mathbf{r} \left\{ \frac{\hbar^2}{2M} |\nabla \Psi|^2 + A|\Psi|^2 + \ldots + G(\Psi + \Psi^*) \right\}, \tag{1}$$

where M is the effective mass of the electron-hole pair, the points show terms of higher order in $|\Psi|^2$, and the terms linear in Ψ violate the law of conservation of the pair number and lift the phase degeneracy.

In the weak-coupling region, the Ginzburg-Landau equation for an excitonic dielectric produced from a semimetal with a sufficiently large band overlap can be derived in the same manner as in the BCS theory for superconductors.

Minimizing (1) at a constant modulus $|\Psi| = \sqrt{n_s}$ (the approximation of an incompressible superfluid component, which is valid for sufficiently small $\nabla\phi$ and G), we obtain for the phase ϕ the sine-Gordon equation

$$\nabla \phi = \frac{\sin \phi}{l^2}, \tag{2}$$

where the length $l = [\hbar^2 n_s^{1/2}/2MG]^{1/2}$ is larger the weaker the phase is fixed.

Equation (2) can be integrated for the one-dimensional problem, when the phase depends only on the single coordinate x (the x direction is chosen along the quasimomentum density). The velocity $v = (\hbar/M)\nabla\phi$ turns out to be a periodic function of x with a period

$$\Delta x = \frac{l}{\sqrt{2}} \int_0^{2\pi} \frac{d\phi}{\sqrt{C - \cos\phi}} = l\sqrt{\frac{2}{C-1}}\, F\left(\pi, \sqrt{\frac{2}{1-C}}\right) \tag{3}$$

which determines the velocity averaged over the volume $v = h/M\Delta x$, with C the integration constant.

The degree of spatial inhomogeneity of the velocity is determined by the ratio of the average velocity $\bar{v}$ to the velocity $w = 4\hbar/Ml$ that characterizes the intensity of the processes that fix the phase. At large velocities ($\bar{v} \gg w$, $\Delta x \ll l$; $C \to \infty$) the periodic component of the velocity is small in comparison with $\bar{v}$, i.e., the phase is less strongly fixed, and the energy E depends on $\bar{v}$ quadratically, just as the kinetic energy of a Galilean-invariant system.

In the other limiting case of low velocities ($\bar{v} \ll w$, $\Delta x \gg l$, $C \to 1$), there are narrow regions of dimension l (solitons), in which the velocity reaches a value on the order of w and the phase is increased by 2π. Outside the solitons, the phase is constant and $v \approx 0$.

We can determine the energy of one soliton:

$$\mathcal{E}_s = \frac{8\hbar^2}{M} \frac{n_s S}{l}, \tag{4}$$

where S is the cross section of the channel through which the electron-hole liquid flows. The total energy is proportional to the number of solitons and therefore depends linearly on $\bar{v}$.

We shall show now that states with nonzero average velocity $\bar{v}$ can be metastable. Let the electron-hole liquid be contained in an angular channel, with the phase shift a multiple of 2π following one circuit through the channel. In this case the total quasimomentum $P = VMn_s$ (where V is the volume of the system) for states that are homogeneous in the transverse direction can assume only quantized values that are multiples of hn_sS, just as the total momentum of a superfluid. The transition from one quantized value of P to another proceeds continuously via vortical states with energy higher than the energy of the initial state that is uniform over the cross section. Thus, energy barriers must be overcome to decrease the quasimomentum P.

If we neglect the processes that fix the phase, then these barriers on the plot of E against the quasimomentum P are identical with the barriers on the plot of the kinetic energy of the superfluid component against the momentum for an ordinary superfluid liquid (see Fig. 2(a) of[8]). The height of these barriers, as is well known, is determined by the maximum of the vortex energy in the moving coordinate system, and increases with decreasing velocity v in proportion to $(1/v)\ln(1/v)$.

The processes that fix the phase influence noticably the height of such barriers only at low velocities $\bar{v} \ll w$. In this case the highest point of the barrier corresponds to vortices of dimension $r \gg l$. The vortex velocity field for Eq. (2) and its critical energy at $r \gg l$ were determined in[8]. Using the results of that reference, we can verify that at $\bar{v} \ll w$ the height of the barrier does not depend on the velocity $\bar{v}$ and is equal in order of magnitude to the height of the barrier without allowance for the fixing of the phase, if $\bar{v}$ is replaced by w in the expression for the barrier.

According to estimates similar to those made for a superfluid (see the references in[8]), the lifetimes of the metastable states considered above are very large so long as $w < v_L$, where $v_L \sim h/M\xi$ is the Landau critical velocity and ξ is the coherence length.

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[1]) According to Kulik and Shevchenko,[7] tunnel transitions, nonetheless, do not exclude the possibility of observing the quantum coherence effect if the phase is "destabilized" by an electric or magnetic field.

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