Dynamics of spherically symmetrical pulsons of large amplitude

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The existence of long-lived ($\tau \sim 10^3$) stable spherically symmetrical pulsons of large amplitude has been observed. Three stages of their evolution are distinguished. Oscillating solitons with n nodes together and the scalar "particle" mass spectrum that they determine are found.

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Classical soliton solutions of relativistically invariant (RI) equations can serve as the basis for the construction of a quantum theory of elementary particles.[1,2] The most interesting of the realistic three-dimensional models of uncharged mesons are not stationary solitons, which are unstable in spherically-symmetrical (ss) geometries,[3] but ss solutions of the oscillator type, obtained in[4,5], which correspond in the (x, t) case to bound states of two solitons.[6,7]

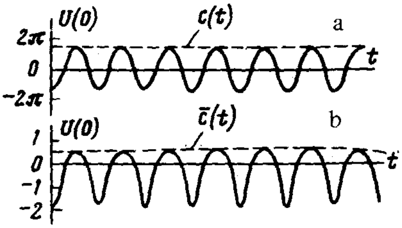

FIG. 1. The $u(0, t)$ curves for Eqs. (1) and (2).

We study in the present paper the formation and further evolution of solitons with large field-oscillation amplitudes $c(t)$ ("heavy" pulsons), and consider the RI nonlinear equations

$$u_{tt} - \Delta_{rr} u + \sin u = 0\,, \tag{1}$$

$$u_{tt} - \Delta_{rr} u - u + u^3 = 0\,. \tag{2}$$

The three characteristic stages in the evolution of the pulsons on Eq. (1) were noted in the course of a computer solution of this equation with initial data (ID) in the form of the (x, t) bion of Eq. (1) (see[6]):

$$u(r,\, 0) = 4\,\mathrm{arc\,tg}\left(\frac{\epsilon}{\omega}\,\mathrm{sech}\,\epsilon r\right) \qquad \omega\,\sqrt{1 - \epsilon^2}\,, \tag{3}$$

where the value $\epsilon/\omega = 10$ was chosen. During the first stage ($t \approx 0$—200), a single-scale field cluster is produced with a bell-shaped oscillating function $u(r, t)$. More than half of the energy $E = \int_0^{r_{\max}} 4\pi r^2 H\, dr$, $H = \frac{1}{2}[u_t^2 + u_r^2 + 2(1 - \cos u)]$ is radiated during this stage. The function $u(0, t)$ (Fig. 1(a)) is seen to be quasi-periodic, with a period $T \sim 7.4$, from practically the very start of the formation process; subsequently, the period decreases slowly (to $T = 2\pi$) together with the decrease of the oscillation amplitude $c(t)$. The amplitude of the produced weak-

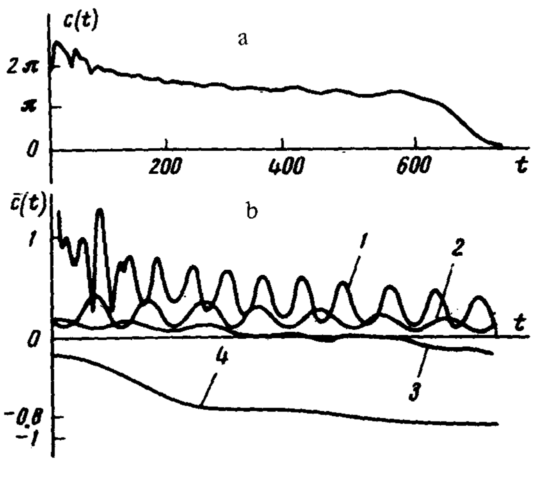

FIG. 2. Time dependence of the pulsons: a) Eq. (1), Eq. (2) (1—$t = 0$—520; 2—$t = 520$—1040; 3—$t = 1040$—1560).

ly-radiating pulson during the second stage ($t \approx 200-630$) is modulated with a period $T_1 \sim 10T$ and decreases slowly from $c(t) \approx 2\pi$ to $c(t) \approx \frac{4}{3}\pi$ (Fig. 2(a)). The evolution of the formed pulson is practically independent of the choice of the initial data $U(r, 0)$ that lead to its formation. The very fact that a long-lived heavy pulson of Eq. (1) is formed from different initial data indicates that it is "stable" in the region $\frac{4}{3}\pi \lesssim c(t) \leq 2\pi$. The same results, on the other hand, allow us to advance the hypothesis that even the sine-Gordon equation, which is fully integrable in the (x, t) case, does not have in the ss geometry stable nonradiating solutions analogous to the plane multisoliton solutions (in particular, solutions describing the bions); see also[4]. During the third stage, starting with $t \approx 630$, a relatively rapid ($\Delta t \sim 80$) decrease of $c(t)$ takes place, to $c(t) \ll 1$, and is accompanied by a "swelling" of the field clusters.

The dynamics of the pulsons of Eq. (2) is similar to that described above. Let us note however, the following differences. The function $u(0, t)$ (its period decreases gradually from $T \approx 5.3$ to $T = \sqrt{2}\pi$) differs in form from a sine function even during the second regular stage ($t \approx 100-1360$) (Fig. 1(b)). Next, immediately after the formation of the weakly-radiating heavy pulson of Eq. (2), the modulation amplitude of the function $\bar{c}(t)$ (the upper envelope of the plot of $u(0, t)$) is relatively large here (Fig. 2(b)). As $\bar{c}(t)$ approaches zero, this amplitude also tends to zero, and the period of the modulation increases monotonically from $T_2 \approx 8T$ at $t \approx 100$ to $T_2 \approx 17T$ at $t \approx 1200$. During the third stage, $\bar{c}(t)$ decreases after $\Delta t \sim 300$ from $\bar{c}(t) = 0$ at $t \approx 1360$ to $\bar{c}(t) \approx -0.8$, when the oscillations become practically symmetrical with respect to the vacuum $u_v = -1$.

Using in ss geometry the method developed in[8], we obtain a denumerable set of small-amplitude pulsons for Eq. (1) (for details see[5])

$$u = u_m A_i(\epsilon r)/A_i(0)\cos(\sqrt{1-\epsilon^2}\,t) + O(\epsilon^3),$$
$$u_m = \sqrt{8}\,\epsilon A_i(0), \qquad u_m^2 << 1. \tag{4}$$

and for Eq. (2):

$$v = 1 + u = v_m A_i\left(\sqrt{\frac{2}{1+\epsilon^2}}\,\epsilon r\right)/A_i(0)\sin\omega t + O(\epsilon^2),$$
$$v_m = \frac{2}{\sqrt{3}}\epsilon A_i(0) << 1, \qquad \omega = \sqrt{\frac{2}{1+\epsilon^2}}. \tag{5}$$

Here $A_i(r) - (i-1)$ are node solutions[9] of the equation

$$A_{rr} + \frac{2}{r}A_r - A + A^3 = 0. \tag{6}$$

The masses of the pulsons (4) and (5), $m_i = E_i$, with different i and equal amplitudes $u_m(v_m)$, are related in the limit $u_m \to 0$ ($v_m \to 0$) like $I_i A_i^{-1}(0)$ ($\approx 1:2:3:4:9:\cdots$), where $I_i = \int_0^\infty A_i^2(r) r^2\,dr$ (cf.[5]). At $u_m \to 0$ ($v_m \to 0$) for the distributions $H_i(r)$ of the pulsons (4) and (5) are stationary, whereas $H(r, t)$ it is essentially nonstationary for the heavy pulsons.

Besides the described heavy nodeless pulsons, we can expect the existence of long-lived heavy pulsons with the nodes of the field function $u(r, t)(v(r, t))$.

The instability of the pulsons of Eqs. (1) and (2) at amplitudes lower than a certain critical value (third stage of the evolution) can be qualitatively attributed to the predominant attraction to the vacuum, which is favored by the boundary conditions.

It is important to note that the pulsons of Eqs. (1) and (2) turned out to be stable to angular perturbations.[1)]

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[1)]Results of two-dimensional calculations, performed with the collaboration of A. B. Shvachka, will be published soon separately.

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