Influence of π condensate on single-nucleon absorption of slow pions by atomic nuclei

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An estimate is presented, showing an appreciable increase of the probability of single-nucleon absorption of slow pions by nuclei under the influence of the π condensate. Measurement of the probability of single-nucleon absorption may be a crucial experiment determining the existence of a condensate in nuclei.

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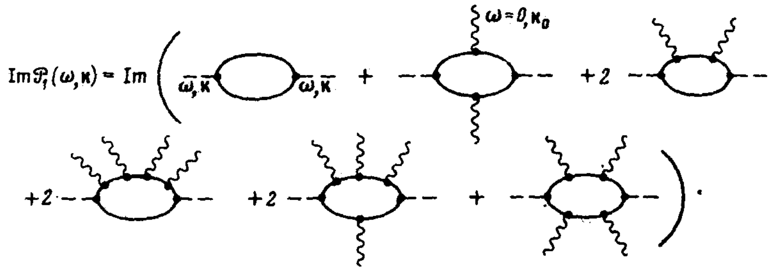
Questions connected with the possible occurrence of pion condensate in nuclear matter has been widely discussed during the last five years. This problem was raised by Migdal,[1] who developed a quantitative theory of the phenomenon and pointed out the possible existence of a condensate at a nuclear-matter density close to that of real nuclei. His theory does not exclude the possible existence of a condensate in real nuclei.[1] Subsequent studies[2] permitted a deeper understanding of the nature of the pion condensate in nuclear matter and in neutron stars. In these studies, the estimated critical density at which the condensate is produced was the same as that in Migdal's first work ($n_c \gtrsim 0.6 n_0$, where n_0 is the normal nuclear density). It is impossible to calculate n_c more rigorously, since it depends strongly on the parameters of the theory. These parameters cannot be theoretically calculated at present, and their values, obtained from the reduction of experimental data on atomic nuclei, have not been determined with sufficient accuracy. The answer to the question whether a condensate exists in a real nucleus can be obtained only by direct experiment. We note that regardless of the results of the experiment the theory makes it possible to determine more rigorously the critical density, and by the same token indicate more definitely the region in which nuclei with anomalous density exist.

An analysis of the known experimental data on the scattering of electrons, M1 l-forbidden transitions, magnetic moments, and the spectra of nuclei and π atoms, carried out in[3–6], does not answer the question of whether a condensate exists in the nucleus, but does not deny the existence of a condensate with amplitude $a^2 \lesssim 0.1 (\hbar = m_\pi = c = 1)$ in real nuclei. It is therefore very important to search for a critical experiment capable of answering this question. In our opinion, such an experiment consists of measuring the probability of single-nucleon absorption of slow pions by atomic nuclei.

It is known that the absorption of slow pions by a single nucleon in infinite homogeneous nuclear matter is strictly forbidden by the conservation laws. The momentum is not conserved in the final system and single-nucleon capture becomes allowed, but to a small degree, so that multinucleon (two-nucleon) absorption is realized. A numerical calculation of the probability of single-nucleo absorption of slow pions by finite nuclei (which determines the partial widths of the π-atom levels), based on the calculation of the imaginary part of the polarization operator in the coordinate representation, leads to a probability $\lesssim 10^{-3}$ for medium and heavy nuclei.[1)]

If π condensate does exist in infinite nuclear matter, the probability of absorption of a slow pion by a single nucleon differs from zero. The onset of the condensate means that the homogeneous nuclear matter is unstable and formation of a wave of spin-isospin density with characteristic dimension k_0^{-1} ($k_0 \gtrsim p_F = 2$) and amplitude $a^2 \lesssim 0.1$ becomes energywise favored. For this reason, the momentum of the nucleon is not conserved and single-nucleon absorption becomes allowed. Let us calculate the imaginary part of the polarization operator $\mathcal{P}_1(\omega, \mathbf{k})$ of a slow pion in infinite nuclear matter, due to single-nucleon absorption with the pion condensate taken into account. Using the expansion of $\mathcal{P}_1(\omega, \mathbf{k})$ in the amplitude of the condensate field up to terms $\sim a^4$, we have



(1)

The wavy line represents the condensate field with wave number k_0, while the solid lines represent the Green's functions of the nucleons. The analytic form of (1) in systems with $N = Z$ is

$$\mathrm{Im}\,\mathcal{P}_1(\omega, \mathbf{k}) = -\frac{2\tilde{f}^2 M p_F}{\pi^2}\mathrm{Im}\,\Phi(\omega,\ 0)k^2 - 8\frac{M p_F}{\pi^2}\,\frac{\tilde{f}^4 k_o^2}{\omega^2[1 + g^-\Phi(0,\ k_o)]^2}$$

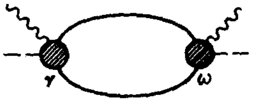
$$\times\left[\mathrm{Im}\,\Phi(\omega,\ k_o)a^2\{\phi^2 - \tfrac{1}{3}\phi_o^2\} + 8\frac{\tilde{f}^2 k_o^2\,\omega^2}{[1 + g^-\Phi(0,\ k_o)]^2\left[\omega^2 - 4\left(\frac{k_o^2}{2M}\right)^2\right]^2}\right.$$

$$\left.\times\ \mathrm{Im}\ \Phi(\omega,\ 2k_o)a^4\phi^2\{2\phi^2 - \tfrac{1}{3}\phi_o^2\}\right]k^2. \qquad (2)$$

In (2) are retained only those terms of a^4 which do not vanish simultaneously with the terms $\propto a^2$. Averaging was carried out over the angle between the vectors $\mathbf{k}$ and $\mathbf{k}_0$. $M = 6.7$ is the mass of the nucleon; $\tilde{f} = f(1-\zeta)$, $f \cong 1$ is the vertex of the πN interaction in vacuum; $\zeta \lesssim 0.2$; $g^- = 1.6$.[7] $\phi = \{\phi_+, \phi_-, \phi_0\}$ is the isovector of the condensate field ($\phi^2 \simeq 1$). At $\omega > \epsilon_F$ the quantity $\mathrm{Im}\Phi(\omega, k)$ differs from zero only in the region $-k^2/2M \le \omega - kv_F \le k^2/2M$ and is equal to

$$\mathrm{Im}\Phi(\omega,\ k) = \pi p_F/4k\left[1 - \left(\omega - \frac{k^2}{2M}\right)^2\Big/(kv_F)^2\right]. \qquad (3)$$

At $k_0 = p_F = 2$, the contribution to $\mathcal{P}_1$ comes from processes $\propto a^4(2)$. Allowance for diagrams more complicated than the skeleton diagrams (1) leads to a significant (by a factor two—three) screening of the terms $\propto a^2$ and $\propto a^4$. Let us explain this with the terms $\propto a^2$ as examples. Taking all the processes into account, these terms can be written in the form

$\mathrm{Im}\,\mathcal{P}(\omega, k) = \mathrm{Im}$ 

. (4)

The main screening in the amplitude $\gamma(\omega = 1, k_0 = 2)$ is due to production of particles with a hole, with total energy and momentum $\omega = 1$ and $k_0 = 2$. This results in the factor $[1 + f\Phi(\omega, k_0)]^{-2}$, where f is the scalar part of the nucleon-interaction amplitudes. Analogously, a factor $[1 + f\Phi(\omega, k_0)]^{-2}[1 + g\Phi(\omega, 2k_0)]^{-2}$ appears in the terms $\propto a^4$. Using (2), we present the lower bound of the effect of the condensate on the single-nucleon absorption. At $k_0 = 2$ we have

$$\mathrm{Im}\,\mathcal{P}_1 \sim -5a^4k^2. \quad (5)$$

We now compare (5), using the connection between the polarization operator and the potential ($2V = \mathcal{P}$), with the imaginary part of the optical potential of slow pions with parameters[8] obtained from experimental data on π atoms

$$\mathrm{Im}\,V = -2\pi n_0^2\{0.04 + 0.08/(1 + {}^{4\pi}/_3\, 0.21 n_0)k^2\}. \quad (6)$$

The second term of (5), which is due to P-absorption of the pion, makes the main contribution to the width of the levels $2p$, $3d$, $4f, \ldots, \pi$ of the atoms. Comparing (5) and (6) we see that at $a^2 = 0.05$ the contribution of the single-nucleon absorption amounts to $\sim 10^{-1}$. The terms of a^2 in formula (2) are different from zero at $2.2 \le k_0 \le 6.2$. For example, at $k_0 = 2.5$ we have

$$\mathrm{Im}\,\mathcal{P}_1 \sim -a^2k^2. \quad (7)$$

In this case the contribution of the single-nucleon absorption becomes ~ 1 ($a^2 = 0.05$). Consequently, estimates for an infinite system increase the probability of single-nucleon emission by 10^2—10^3 times if a π condensate exists with amplitude $a^2 = 0.05$. These estimates of the influence of the condensate on the probability of single-nucleon absorption seem reasonable, since the characteristic momentum is $k_0 \gtrsim p_F \gg p_F/A^{1/3}$. In addition, it appears that if account is taken of the finite dimensions of the nucleus, the contribution of the terms $\propto a^2$ to $\mathcal{P}_1$ will differ from zero already at $k_0 = p_F = 2$, owing to the nonconservation of the nucleon momentum as a result of the finite dimensions; this will raise somewhat the lower bound of the foregoing estimate. In the case of absorption of stopped π^- mesons, allowance for the correct pion wave functions satisfying the Klein—Gordon equation in the total potential alters the result insignificantly. Estimates show that the probability of the single-nucleon capture of slow pions is greatly enhanced if a π condensate exists in the nucleus (by $> 10^2$ times at $a^2 \approx 0.05$). Measurements of the probability of the emission of a nucleon of energy in the range 140 MeV will apparently provide the answer to the question of the existence a condensate in real nuclei.

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[1)]In the calculation we used π^--meson wave functions satisfying a Klein-Gordon equations in a Coulomb potential and a strong-interaction potential. The details of the calculations will be reported in a separate paper.

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