Evaporation of black miniholes; high-energy physics

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The observation of the final stages of the evaporation of black miniholes, if they should be discovered, might provide important information on the physics of high energies, up to the Planck level. Some relevant estimates are offered. It is shown, in particular, that the "shadow world" which has been proposed in certain theories might play a role in the dynamics of the evaporation.

Most of the processes and features of the physics of unified theories of all types of interactions, especially those incorporating gravity, will be beyond study under laboratory conditions with the help of accelerators for the foreseeable future. One way to test the theories is to apply them to the early cosmology and to make comparisons with observational data. In the present letter we wish to call attention to some possibilities related to the observation of the final stages of the quantum evaporation of black miniholes—if they should be discovered. The hypothesis of the possible formation of miniholes in an early stage of the expansion of the universe was predicted in 1966 by Zel'dovich and Novikov[1] and also, independently, by Hawking.[2] Hawking subsequently showed[3] that black holes are radiators with the temperature

$$T = \frac{M_p^2}{8\pi M}, \tag{1}$$

where M is the mass of the hole, M_p is the Planck mass, and we are setting $k = \hbar = c = 1$. Some qualitative arguments which lead to an expression like (1) were offered by Bekenstein.[4] So far, miniholes (and their radiation) have not been observed, and estimates of their possible abundance in "inflationary" cosmological scenarios are not promising. Nevertheless, a search for them would not be hopeless.

In the final stages of the evaporation, the temperature T rises to the extent that particles of very large mass, up to Planck's mass, could be emitted. In principle, there is a small probability that even larger particles, including magnetic monopoles and strings—if entities of these two types exist—could be emitted.

The evaporation dynamics depends on the effective number of species of evaporated particles; we will denote this number by g. The rate of the total evaporation of energy is

$$\left|\frac{dM}{dt}\right| = \frac{g}{2}\,\frac{dE}{dt}, \tag{2}$$

where dE/dt is the radiation of energy by photons.

Some of the theories which are presently being discussed hypothesize a "shadow world." This shadow world would consist of particles which interact with "our own world" only through gravitation. The particles of this shadow world should be evaporated from a black hole along with "our own" particles, and they should double the rate of change of the mass of the black hole (i.e., the value of g). If symmetry breakings occur in this shadow world in a way different from the way in which they occur in our own world, as is suggested in many versions of the theory, and if the particles remain massless, then at black-hole temperatures corresponding to the masses of our particles or less, the rate of change of the mass would change by a factor of several tens or even several hundreds. A test of the suggested existence of a shadow world is one possible application of the evaporation of black holes in high-energy physics.

Let us look at some estimates relevant to methods of determining the parameter of g and also some estimates regarding the formation of particles of large mass.

At low temperatures, the emission of particles and the loss of mass as a result of this emission are described by

$$\frac{dn}{dt} = \frac{1}{2\pi^2} \int_0^\infty \frac{Sp^2 v}{e^{E/T} \mp 1} dp, \tag{3}$$

$$\frac{dM}{dt} = - \frac{1}{2\pi^2} \Sigma \int_0^\infty \frac{Sp^3}{e^{E/T} \mp 1} dp . \tag{4}$$

These expressions are written for a nonrotating black hole; p is the momentum, v is the velocity, E is the energy of the emitted particles, and S is the cross section for the capture of a particle by the gravitational field of a black hole.

As the temperature increases in the course of the evaporation of the black hole, progressively heavier particles participate in the evaporation, as is manifested by an increase in the value of g. With a further increase in the temperature, the interactions of particles and the phase transitions of the vacuum associated with the restoration of symmetries broken at a low temperature become important. At energies and temperatures comparable to Planck's energy M_p, effects of "extra dimensions" (if they exist) may be important, and the emission process may not be of a thermal nature. All of these phenomena, like the existence of a shadow world, should be manifested in the value of and in the changes in g.

At a wavelength $\lambdabar \ll r_g$ of the emitted particles, where $r_g = 2M/M_p^2$ is the gravitational radius, the capture cross section is

$$S = \frac{27\pi r_g^2}{4} \left(\frac{1}{3} + \frac{2}{3v^2} \right) . \tag{5}$$

In the important momentum region, however, we have $\lambdabar \sim r_g$, and S is smaller than the value given by (5). For estimates below we assume

$$S = \pi r_g^2 \xi, \qquad \xi = \text{const} \cong 1.9.$$

This particular value of ξ has been chosen to satisfy the results of Page's numerical calculations for photon emission.[5]

With $g = \text{const}$, the following quantity is constant:

$$-M_p^2 \frac{d}{dt} (T^{-3}) \approx 0.93g. \tag{6}$$

A determination of this quantity or of the similar and perhaps more convenient integral combination (t_0 is the time at which the process ends)

$$\frac{M_p^2}{T^3(t_0 - t)} \approx 0.93g \tag{7}$$

would make it possible to determine g [in the case in (7), the value would be averaged in a certain way]. Instead of T, we might use a quantity which is proportional to it (the energy of the spectral maximum or the average energy of the photons; it must be kept in mind here that the spectrum is non-Planckian because of the $\hbar$ dependence of S).

At the end of the evaporation, it would become difficult to time-resolve the process, and it would probably be better to use data on the spectrum integrated over time. We denote by $N(E_0)$ the total number of emitted photons with energies $E > E_0$. Under our assumptions ($\xi = \text{const}$, $g = \text{const}$) we would then have

$$\frac{M_p^2}{E_0^2 N(E_0)} \approx 2\pi \frac{\zeta(4)}{\zeta(5)} g = 6.6g, \tag{8}$$

where ζ is the Riemann zeta function [the derivation of this result is analogous to the derivation (given just below) of expression (9)].

In the case $g \neq \text{const}$, this equation would again give us an average value.

To test a particular theory, we would of course have to calculate the functions (6)–(8) theoretically and compare them with observational data.

Let us find the number of scalar particles of mass m which are produced in the evaporation of a black hole. To find estimates, we work from (3) and (4), assuming $g = \text{const}$ and $\xi = \text{const}$. Dividing (3) by (4), we find, for particles with charges of a common sign ($\epsilon = E/T$, $\mu = m/T$),

$$\frac{dn}{dM} = \frac{4\pi M}{3g\zeta(4) M_p^2} \int_\mu^\infty \frac{\epsilon^2 - \mu^2}{e^\epsilon - 1} d\epsilon,$$

using the substitution $M = M_p^2 (8\pi m)^{-1} \mu$, we find

$$n = \frac{M_p^2}{48 g m^2 \zeta(4)} \int_0^\infty \mu d\mu \int_\mu^\infty \frac{\epsilon^2 - \mu^2}{e^\epsilon - 1} d\epsilon.$$

The double integral is evaluated by switching the order of integration; the result is $6\zeta(5)$. We thus find (for two signs of the charge)

$$2n = \frac{M_p^2 \zeta(5)}{4\pi g m^2 \zeta(4)}. \tag{9}$$

For spin-1/2 particles we have an additional factor of 15/8. The incorporation of the factor $1/3 + 2/(3v^2)$ from (5) would lead to an additional factor of 5/3.

The observation of the production of magnetic monopoles and strings would be of particular interest. The probabilities for these processes, however (under the assumption that monopoles and strings do exist), would not be described by estimate (9), and they would be very small. For monopoles we would have to take their nonlocal nature into consideration, as was pointed out to me by A. D. Linde. The scale dimension of monopoles, $\rho \sim (g^2 m)^{-1}$ (here $g < 1$ is the coupling constant), is much larger than the gravitational radius of a black hole,

$$r_g = \frac{2M}{M_p^2} = \frac{1}{4\pi T}$$

at a temperature $T \sim m$. In the region of importance to estimate (9), therefore, the cross section (S) for the capture of monopoles by a black hole is very small, $S \ll \pi r_g^2$, and we find $\xi < 1$. We would have $\rho \sim r_g$ and $\xi \sim 1$ only at $T \ll m$. As a result, the number of monopoles which form would be much smaller than that given by estimate (9).

To estimate the probability for the production of strings, the circumstance that their mass (M_s) is very large, on the order of M_p, is particularly important. For example, we have $m_s = 2M_p$. A string could be emitted by a black hole only if its initial mass satisfied $M_0 > m_s$. In this case, the temperatures of the initial state would be, respectively,

$$T_0 < \frac{M_p^2}{8\pi m_s} = \frac{M_p}{16\pi} \quad \text{or} \quad T_0 < \frac{m_s}{32\pi} \ll m_s .$$

Actually, a quasiequilibrium thermal analysis would be completely inapplicable in the emission of particles with energies comparable to the initial mass of the black hole. I have not been able to find a suitable method for finding an estimate. It can apparently nevertheless be asserted that under the condition $T_0 \ll m_s$ the probability for the emission of a string is very low. The nonlocal nature of a string leads to a further decrease in the probability. In the theories which have been proposed, the string is an unstable particle, decaying into local particles in a time on the order of Planck's time.

A detailed study of the processes that occur at energies $E \sim M_p$ would probably be possible only on a close fly-by of a black hole at the very end of its existence by an unmanned experimental space probe launched into deep space especially for the purpose. In other words, such a detailed study would be possible only in the remote future of the cosmic era. On the other hand, we do not rule out the possibility that other methods for direct experimental study of energies at the Planck scale will have become available.

So far, we have ignored effects of a rotation of the black hole. We denote by J the total angular momentum of the rotation. We introduce the relative rotational angular momentum $0 < j < 1$, setting $J = M^2/M_p^2 j$. At $j \ll 1$, we can ignore the rotation. At $j \sim 1$, "superradiation" effects will play a key role (Zel'dovich,[6,7] Starobinskiĭ,[8] and Curir[9]).

On the basis of dimensionality considerations ($C \sim 1$) we would write

$$\frac{dJ}{dt} = Cj \frac{M}{M_p^2} \frac{dM}{dt} .$$

At $C > 2$, there would be a decrease with decreasing M. The coefficient would depend on j and M. Assuming that we have $C = \text{const}$ near $j = 0$, we find $j \sim M^{C-2}$; i.e., in the case $C > 0$, the path $j = 0$ is stable. Calculations of the emission of angular momentum and mass for a rotating black hole which were carried out by Page[10] show that C is substantially greater than 2 for particles of various spins over the entire range of j. These calculations were carried out for massless particles. Incorporating effects of a mass and certain refinements of superradiation effects according to Ref. 9 apparently do not alter Page's qualitative conclusion that j falls off rapidly as the black hole evaporates. The value of j and the spatial orientation of the rotation axis can be determined from observations of the circular and linear polarization of the photons.

In summary, we have offered estimates of some effects of the evaporation of black holes which would be of interest to high-energy physics. The results show that a study of the temperature of the emitted particles as a function of the time and of the spectrum of the particles could furnish information on the existence of a shadow world and on the characteristic features of the theory at the very highest energies, including the grand-unification energy and Planck's energy. We have offered an estimate on the production of particles with a finite mass, and we have discussed the production of monopoles and strings.

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