Exclusive structure functions of the constituents in hadrons

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An exact solution expressing the exclusive structure functions in terms of the inclusive functions for pions and protons can be obtained if the latter are regarded as consisting of a quark and a diquark and if it is assumed that the independence of the valence constituents in hadrons is limited solely by the conservation of total energy. The method proposed here can be extended in a straightforward manner to other hadrons.

In many problems arising in the study of the interaction of hadrons it is important to know not only the inclusive structure functions but also the exclusive structure functions which describe the distribution of all the partons in the hadron at once or at least those partons which are most important for a given process. This situation arises in virtually any attempt to go beyond the limits of the impulse approximation (i.e., a one-gluon exchange with a soft discoloration) in quantum chromodynamics. We do not claim to solve the problem completely, since a complete solution would essentially imply the derivation of a complete quantum chromodynamic hadron theory. In the case of a valence quark and antiquark in a meson and a quark and diquark in a baryon,[1)] however, the search for exclusive distributions from the given inclusive structure functions turns out to be surprisingly simple if it is assumed that the independence of these constituents in the hadrons is limited solely by the conservation of the total energy.[2)] We will discuss this subject using a pion and proton as an example, since the generalization to other hadrons can easily be made.

We will first consider a pion as a very simple system. For definiteness, we assume that it is a π^0 meson (analysis of charged mesons is the same in every respect, since the inclusive structure functions of both quarks in them are the same). For definiteness, we also assume that this meson consists of a u quark and a $\bar{u}$ quark plus all possible configurations of gluons and sea particles, the integral over which is 1. The last hypothesis is, however, justifiable in all cases except in a very narrow interval $1-x-y<\delta\ll 1$, because the energy distribution of all these particles is close to zero.

In this approximation we have the equation

$$\int_0^1 \varphi(x)\varphi(y)\theta(1-x-y)\,dy = u(x), \tag{1}$$

where $u(x)$ is the inclusive structure function, and the product $\varphi(x)\varphi(y)\vartheta(1-x-y)$ is the exclusive distribution of the two quarks with x and y fractions of energy, respectively. Integrating both parts over x from $x=0$ to $x=1-t$ and changing the order of integration over x and t, we find

$$\int_0^t \varphi(y)dy \int_0^{1-t} \varphi(x)dx + \int_t^1 \varphi(y)dy \int_0^{1-y} \varphi(x)dx = \int_0^{1-t} u(x)dx. \tag{2}$$

Expressing the integrals over x on the left side of (2) by means of Eq. (1), we find a simple equation for the function φ

$$\int_0^t \varphi(y)dy = \frac{\varphi(t)}{u(t)} \int_t^1 [u(1-x)-u(x)]\,dx \equiv \varphi(t)R^{-1}(t), \tag{3}$$

which can be solved immediately by differentiating over t, since $R(t)$ is a known function:

$$R(t) = u(t)\{\int_t^1 [u(1-x)-u(x)]\,dx\}^{-1} \equiv u(t)\{\int_0^t [u(x) - u(1-x)]\,dx\}^{-1}. \tag{4}$$

We finally find

$$\varphi(t) = CR(t)\exp\int R(t)dt, \tag{5}$$

where C is the normalization constant. In the given approximation the exclusive distribution of the two valence quarks in the pion thus has the from[3)]

$$\varphi(x)\varphi(y)\theta(1-x-y) = C^2R(x)R(y)\theta(1-x-y)\exp\{\int R(x)dx + \int R(y)dy\}. \tag{6}$$

We call attention here to the fact that Eq. (5) is a solution of Eq. (3) only if the substitution of the lower limit on the left side of (3) reduces the original substitution to zero. The reason for this condition is that the corresponding boundary information in (3) was lost as a result of differentiation over t. The fact that the general solution of (5) thus obtained does not contain an additive arbitrariness means that Eq. (3) [and hence Eq. (1)] can generally be solved only in the case indicated above, since the class of solutions can only expand as a result of differentiation. Using expresion (4) for the function $R(t)$, we easily see that the solvability condition formulated above can be written in the form

$$\lim_{t\to 0}\left[\left(1-\frac{u(1-t)}{u(t)}\right)\ln t\right] = -\infty. \tag{7}$$

This essentially means that

$$\lim_{t\to 0}\frac{u(1-t)}{u(t)} < 1. \tag{7a}$$

The physical meaning of this constraint is simple: The configuration in which the valence quark has a small fraction of the total hadron energy probably leaves a large phase volume for all the remaining partons, so that it should have a higher probability than the configuration in which this fraction of the quark contains nearly the total energy and the phase volume mentioned above is compressed to the limit. The obviousness of this line of reasoning, in our view, gives it reasonably general nature, so that condition (7a) should not be considered a consequence of the assumptions upon which Eq. (1) is based; this condition most likely shows that this equation can always be solved for real mesons.

As for the protons, under the assumption that they have a quark-diquark structure, the corresponding exclusive distribution can be determined from the following system of equations.

$$D(x)\int_0^{1-x} dy \int_0^{1-x-y} F(y+z)\,dz = d(x)$$
$$\int_0^{1-x} F(x+y)\,dy \int_0^{1-x-y} D(z)\,dz = u(x), \qquad (8)$$

where $u(x)$ and $d(x)$ are the inclusive structure functions of the u and d quarks, and $D(x)F(y+z)\theta(1-x-y-z)$ is the exclusive structure function of the d quark with x fraction of energy and of the diquark consisting of two u quarks with y and z fractions of energy, respectively.

Using the notation $y+z=\omega$ in the first equation, introducing in the second equation the variable $\xi = y + x$ instead of y, and differentiating the second equation with respect to x we find

$$D(x)\int_0^{1-x} F(\omega)\,\omega\, d\omega = d(x)$$

$$F(x)\int_0^{1-x} D(z)\,dz = -\frac{du(x)}{dx}. \qquad (8a)$$

If we now integrate, say, the first equation in (8a) over x from $x=0$ to $x=1-t$ and then change the order of integration over x and t and write the inner integral over x from the second equation [in complete analogy with the manner in which this was done above in the case of Eq. (1)], we find the equation for the function F. We will not write out this equation because it is similar to Eq. (3). The solution of this equation is

$$F(x) = CK_1(x)\exp\int K_1(x)\,x\,dx, \qquad (9)$$

where

$$K_1(x) = \frac{du(x)}{dx}\left\{\int_0^x \left[y\,\frac{du(y)}{dy} + d(1-y)\right]dy\right\}^{-1}. \qquad (10)$$

In a corresponding way, we find for the function D an expression which is the same as (5), in which we need only replace $t(x)$ by the function

$$K_2(x) = d(x)\left\{\int_0^x \left[d(y) - (1-y)\,\frac{du(1-y)}{dy}\right]dy\right\}^{-1}.\ ^{4)} \qquad (11)$$

Making use of the asymptotic behavior of the functions[4] $u(y)$ and $d(y)$: $u(y) \sim d(y) \sim y^{-\alpha R^{(0)}}$ in the limit $y \to 0$, and $u(y) \sim (1-y)^{\alpha R^{(0)} - 2\alpha N^{(0)}}$ and $d(y) \sim (1-y)^{\alpha R^{(0)} - 2\alpha N^{(0)} + 1}$ in the limit $y \to 1$, we see that the solution which we found satisfies system (8a), since the values of the corresponding original solutions at the lower limit of integration automatically vanish [we recall that $\alpha_R(0) \cong 0.5$ and $\alpha_N(0) \cong -0.4$].

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1) Such an analysis of the three-quark systems is based on extensive evidence both of theoretical[1] and phenomenological[2] nature.
2) This assumption can be justified by the fact that the interaction between quarks is strong only at long range (on the order of the effective hadron radius), i.e., just when essentially the entire mass (energy) of the hadron is contained in the quarks because of the increase in the holding potential and hence the quark mass.[3]
3) This distribution is at the same time an inclusive distribution over all remaining (nonvalent) partons.
4) In relations (10) and (11) we took into account that in the proton even the normalization condition $u(1) = 0: \int^1 u(y)dy = \int_0^1 d(y)dy = 1$.

[1]A. Martin, Prepring CERN–Th 4295/85.
[2]A. Capella, V. Sukhatme, and J. Tran Thanh Van, Z. Phys. **C3**, 329 (1980); A. Capella and J. Tran Thanh Van, Phys. Lett. **114B**, 450 (1982); A. B. Kaĭdalov and K. A. Ter-Martirosyan, Yad. Fiz. **39**, 1545 (1984) [Sov. J. Ncul. Phys. **36**, 979 (1984)].
[3]A. De Rujula, H. Georgi, and S. L. Glashow, Phys. Rev. D **12**, 147 (1975); D. B. Lichtenberg, W. Namgung, E. Predazzi, and J. G. Wills, Phys. Rev. Lett. **48**, 1653 (1982); I. V. Andreev, Yad. Fiz. **41**, 1345 (1985) [Sov. J. Nucl. Phys. **41**, 855 (1985)].
[4]A. B. Kaĭdalov, Yad. Fiz. **33**, 1369 (1981) [Sov. J. Nucl. Phys. **33**, 733 (1981)].

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