Effect of the order parameter dynamics on the phonon emission in superconductors

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A fluctuating phonon flux from the "phase-slip centers" is predicted. The presence of such a flux would confirm the basic concepts of the dynamic theory of the resistive state.

1. At $T \sim T_c$ the dynamic equation[1–5] for the order parameter Δ

$$-\frac{\pi}{8T\sqrt{1+(2\tau_\epsilon|\Delta|)^2}}\left[\frac{\partial}{\partial t}+2i\varphi+2\tau_\epsilon^2\frac{\partial|\Delta|^2}{\partial t}\right]\Delta+\frac{\pi}{8T_c}D(\vec{\nabla}-2i\mathbf{A})^2\Delta$$

$$+\left[\frac{T_c-T}{T_c}-\frac{7\zeta(3)}{8(\pi T)^2}|\Delta|^2\right]\Delta=0 \tag{1}$$

contains the energy relaxation time of electrons, τ_e, which may depend on time under nonequilibrium conditions. Assuming that the energy decay of electrons $\gamma = 2/\tau_\epsilon$ is due primarily to the inelastic collisions with actual phonons, we represent γ in the form

$$\gamma \approx \frac{7\pi\lambda\zeta(3)T^3}{(up_F)^2}+\frac{\pi\lambda}{(up_F)^2}\int_0^\infty d\omega_q\,\omega_q^2\,\delta N_{\omega_q}=\gamma_0+\delta\gamma, \tag{2}$$

where $\delta N_{\omega_q} = N_{\omega_q} - N^0_{\omega_q}$ is the nonequilibrium part of the phonon distribution function, u is the speed of sound, and λ is the dimensionless electron-phonon coupling constant. The quantity $\delta N_{\omega_q}(t)$ can be determined from the kinetic equation for phonons,

$$\frac{d}{dt}(\delta N_{\omega_q})=J(N_{\omega_q})+L(N_{\omega_q}), \tag{3}$$

where $J(N_{\omega_q})$ is the phonon-electron collision integral, whose explicit form is given in Ref. 6, and $L(N_{\omega_q})$ is an operator which describes how the phonons are related to the environment (the heat sink). In the approximation of Ref. 7, this operator can be written

$$L(N_{\omega_q})\approx-\delta N_{\omega_q}/\tau_{es}, \tag{4}$$

where $\tau_{es} \sim d/u$, and d is the scale dimension of the superconductor. The phonon-electron inelastic collision integral can be simplified considerably in the approximation of the "local equilibrium" between the condensate and the single-electron excitations. The single-electron excitations in this case are described by the functions[1-5]

$$f_1(\epsilon) \equiv (1 - n_\epsilon - n_{-\epsilon}) \operatorname{sign} \epsilon \approx -\tau_\epsilon \frac{\partial |\Delta|}{\partial t} \frac{R_2}{N_1} \frac{\partial f_0}{\partial \epsilon} + f_0(\epsilon), \quad f_0(\epsilon) = \tanh \frac{\epsilon}{2T},$$

$$f_2(\epsilon) \equiv -(n_\epsilon - n_{-\epsilon}) \frac{\operatorname{sign} \epsilon}{N_1} \approx \frac{N_1 (\partial f_0 / \partial \epsilon)\varphi + \tau_\epsilon |\Delta| N_2 (\partial \theta / \partial t)(\partial f_0 / \partial \epsilon)}{2\tau_\epsilon |\Delta| N_2 + N_1}; \tag{5}$$

$$N_1 = \operatorname{Re} \frac{\epsilon + i\gamma}{\sqrt{(\epsilon + i\gamma)^2 - |\Delta|^2}}, \quad N_2 = -\operatorname{Im} \frac{|\Delta|}{\sqrt{(\epsilon + i\gamma)^2 - |\Delta|^2}},$$

$$R_2 = \operatorname{Re} \frac{|\Delta|}{\sqrt{(\epsilon + i\gamma)^2 - |\Delta|^2}}. \tag{6}$$

Substituting (5) into the expression[6] for $J(N_{\omega_q})$ and taking account of the fact that the quantities u_ϵ and v_ϵ used in Ref. 6 become N_1 and R_2, respectively, we find

$$J(N_{\omega_q}) \approx \frac{\pi\lambda}{2} \frac{\omega_D}{\epsilon_F} \left\{ 2 \frac{\partial |\Delta|}{\partial t} \frac{\tau_\epsilon}{T} N^0_{\omega_q} \eta_1 - \delta N_{\omega_q} \eta_2 \right\}, \tag{7}$$

$$\eta_1 = \int_0^\infty d\epsilon \frac{P(\epsilon) R_2(\epsilon)}{\cosh^2(\epsilon/2T)} + \int_0^\infty d\epsilon Q(\epsilon) \left\{ \frac{R_2(\epsilon + \omega_q)}{N_1(\epsilon + \omega_q)\cosh^2[(\epsilon + \omega_q)/2T]} - \frac{R_2(\epsilon)}{N_1(\epsilon)\cosh^2(\epsilon/2T)} \right\}, \tag{8}$$

$$\eta_2 = \int_0^{\omega_q} d\epsilon P(\epsilon) \tanh \frac{\epsilon}{2T} + \int_0^{\omega_q} d\epsilon Q(\epsilon) \left(\tanh \frac{\epsilon + \omega_q}{2T} - \tanh \frac{\epsilon}{2T} \right), \tag{9}$$

and

$$P(\epsilon) = N_1(\epsilon) N_1(\omega_q - \epsilon) + R_2(\epsilon) R_2(\omega_q - \epsilon),$$

$$Q(\epsilon) = N_1(\epsilon) N_2(\omega_q + \epsilon) - R_2(\epsilon) R_2(\omega_q + \epsilon). \tag{10}$$

The function η_1 for various parameters of the superconductor is plotted in Fig. 1. Under the same conditions the function η_2 is nearly linear: $\eta_2 \approx c\omega_q$, where $c \approx 1$.

From the relations which we obtained we can find $\delta N_{\omega_q}(t)$ and establish a relationship between the order parameter dynamics and the nonequilibrium phonons.

2. We define the "generalized local-equilibrium approximation (equilibrium between the condensate, the electronic excitation, and the phonons) as the approxima-

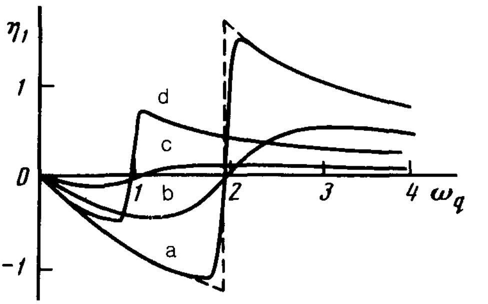

FIG. 1. The function $\eta_1(\omega_q)$ at $T=5$. a) $\Delta=1$, $\gamma_0=0.01$; b) $\Delta=1$, $\gamma_0=0.3$; c) $\Delta=0.5$, $\gamma_0=0.3$; d) $\Delta=0.5$, $\gamma_0=0.01$. The dashed line is the case in which $\Delta=1$ and $\gamma_0=0$. All quantities are given in units of Δ_0.

tion in which the "local equilibrium" conditions are satisfied[1-5] and in which it is assumed that the characteristic frequencies (and the wave vectors) at which N_{ω_q} varies are low in comparison with $\lambda\omega_D T/\epsilon_F$, so that the left side of Eq. (3) can be dropped. From (3)–(10) in this case we find for δN_{ω_q}, which depends on **r** and t only in an implicit manner [through $\Delta(\mathbf{r},t)$], the expression which we write in the limit $\tau_{es}\to\infty$, in which the nonequilibrium nature of the phonon system is particularly well defined:

$$\delta N_{\omega_q} = \frac{\partial|\Delta|}{\partial t} N^0_{\omega_q} \frac{\eta_1}{\eta_2 T\gamma_0} . \qquad (11)$$

Substituting (11) into (2), we find

$$\delta\gamma \sim \frac{1}{T}\frac{\partial|\Delta|}{\partial t} . \qquad (12)$$

Since the characteristic frequencies at which $|\Delta|$ changes in this case are small compared with γ_0 [the condition under which Eq. (1) can be used[1-5]], we have $\delta\gamma/\gamma_0 \ll 1$ even in the limit $\tau_{es}\to\infty$. The nonequilibrium nature of the phonon system in the "generalized local-equilibrium" approximation therefore affects the behavior of the order parameter only slightly. This parameter can be described by Eq. (1) with τ_ϵ which does not depend on time. [We note that the phonons may increase in importance if the conditions of the approximation mentioned above are violated. Equations (1) and (3) in this case must be considered jointly.]

3. We now consider the limiting case $\tau_{es}\to 0$ in which, according to (4), $\delta N_{\omega_q}\to 0$. This condition is satisfied when $d \lesssim \xi(T)$ (for a superconducting film or filament, for example). The emission of phonons from a superconductor into the heat sink in this case is given by (7). According to Ref. 6, the intensity of the phonon flux emitted by a volume ϑ in the spectral interval $d\omega_q$ is

$$dW_{\omega_q} = J(N^0_{\omega_q})\rho(\omega_q)d\omega_q, \quad \rho(\omega_q) = \vartheta\,\omega_q^3/2\pi^2u^3 . \qquad (13)$$

It follows from (1), (7), and (13) that any change in the order-parameter modulus is accompanied by an exchange of phonons between the superconductor and the heat sink.

As an example, we will consider a situation which occurs in narrow superconducting filaments or whiskers which are in the resistive state. According to the dynamic model (see, e.g., Ref. 8), these states are characterized by periodically spaced "phase-slip centers." The order-parameter modulus periodically vanishes at these centers with a scale length $\Lambda \sim \xi(T)[\gamma_0/\Delta_0(T)]^{1/2}$, where Δ_0 is the equilibrium value of the gap. The oscillation frequency is determined by a Josephson-type relation: $\omega = 2V$, where V is the potential difference at the center.

Expressions (13), (7), and (8) can be used to calculate the spectral dependence of the phonon emission from the phase-slip center at a specified time. Qualitatively, this dependence is similar to that shown in Fig. 1; at $\omega \gg T$ the emission is small. Since $|\Delta(t)|$ is a periodic function of time,[5] the phonon emission from the active region is pulsating in nature and the phonon flux periodically reverses its direction, as is evident from the spin-changing factor $\partial|\Delta|/\partial t$ in (7) and (13).

The intensity of the phonon flux per unit volume is $w \sim (\omega_D/\epsilon_F)(\Delta^2/\gamma)(T^3/u^3)V$. For $V < \gamma$ (for example, $V \sim 10^2$ nanovolts) at $T \sim 10$ K we would have $w \sim 10^3$ W/cm^3, which is several orders of magnitude greater than the ohmic dissipation, $p \sim V^2/\rho\Lambda^2$, where ρ is the resistivity of the active region of length Λ.

An alternating pulsating flux is not easy to detect. One approach would be to use a high time resolution technique. Another approach would be to detect the phonon flux. This method should also be sensitive to the direction of the phonon flux (for example, a method based on the ^{4}He fountain effect[9] could be used).

A phonon emission pattern spatially modulated and periodic with respect to time would constitute a direct confirmation of the validity of the current theoretical understanding of the physics of the resistive state.[8] Of considerable interest in this connection are the flat films in the resistive state, which typically exhibit the phase-slip lines.[10] A detailed information about the shape of these lines and about their time evolution could be obtained from the analysis of their characteristic phonon emission.

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