Undamped helicon

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The spectrum of low-frequency electomagnetic waves in a superlattice is calculated for the case with a quantum Hall effect.

The experimental observation[1] of a quantum Hall effect in a GaAs/(AlGa)As superlattice in a magnetic field directed perpendicular to the layers of this lattice makes it an urgent matter to study the electrodynamic properties of such systems. A superlattice in a quantizing magnetic field differs from a two-dimensional system in its conductivity along the magnetic field $\mathbf{H}_{ex} = \vec{\eta} H$. The Hall conductivity σ_H has a quantized value $e^2 s/2\pi\hbar d$ at $H = H_s = (2\pi e\hbar c n d)/s$ (d is the period of the superlattice, n is the *volume* density of electrons in the superlattice, and s is an integer) when the Fermi energy of the carriers[1)] coincides with the boundary of one of the minibands, and the electron system thereby simulates an insulator, but one in which a nondissipative Hall motion of electrons is allowed.[2,3]

At $H = H_s$ all three dissipative components of the conductivity are zero ($\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = 0$) or at least very small in comparison with the Hall conductivity σ_H. In those cases in which the Hall conductivity is substantially greater than the dissipative conductivity, weakly damped waves (helicons[4]) can propagate through the

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conductor. In a superlattice, helicons should have some unusual properties, because of the vanishing of all the dissipative components of the conductivity.[2)]

The Hall current is $\mathbf{j}_H = \sigma_H [\vec{\eta}\mathbf{E}]$. Assuming, for simplicity, that the dielectric constant of the superlattice is isotropic, we write Maxwell's equations in the $\mathbf{k}\omega$ representation ($\mathbf{k}$ is the wave vector, and ω the frequency):

$$[\mathbf{kH}] = -(4\pi i\sigma_H/c)\,[\vec{\eta}\mathbf{E}] - (\omega\epsilon/c)\mathbf{E},$$
$$[\mathbf{kE}] = (\omega/c)\mathbf{H}, \qquad (1)$$

where $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic fields of the wave. By using static values of σ_H and ϵ we are restricting the analysis to low-frequency, long-wavelength oscillations. In particular, we have $\omega \ll \omega_c$, where ω_c is the cyclotron frequency in the field H. The frequency ω may exceed $4\pi\sigma_H = 4\pi ne^2/m^*\omega_c = \omega_L^2/\omega_c$ (provided, of course, that the condition $\omega_c \gg \omega_L$ holds; m^* is the effective mass of the conduction electrons). From (1) we have

$$[\vec{\eta}\mathbf{E}](\omega^2\epsilon/c^2 - k^2) + (4\pi i\sigma_H/\omega\epsilon)[\mathbf{k}\vec{\eta}]\,(\mathbf{k}[\vec{\eta}\mathbf{E}]) - (4\pi i\sigma_H\omega/c^2)[[\vec{\eta}\mathbf{E}]\vec{\eta}] = 0;$$
$$(\mathbf{E}\vec{\eta}) = (4\pi i\sigma_H/\omega\epsilon)(\omega^2\epsilon/c^2 - k^2)^{-1}(\vec{\eta}\mathbf{k})(\mathbf{k}[\vec{\eta}\mathbf{E}]). \qquad (2)$$

It is thus a simple matter to derive a dispersion relation relating ω and $\mathbf{k}$:

$$(k^2 - \omega^2\epsilon/c^2)^2 - (4\pi\sigma_H/\omega)^2\,(\omega^4/c^4 - \omega^2k^2\sin^2\theta_k/c^2\epsilon) = 0. \qquad (3)$$

Here θ_k is the angle between the vectors $\vec{\eta}$ and $\mathbf{k}$; this angle specifies the wave propagation direction. From (3) we have

$$k^2 = \frac{\omega^2}{c^2}\epsilon - \frac{1}{2}\left(\frac{4\pi\sigma_H}{c}\right)^2\frac{\sin^2\theta_k}{\epsilon} \pm \sqrt{\left(\frac{4\pi\sigma_H\omega}{c^2}\cos^2\theta_k\right)^2 + \frac{1}{4}\left(\frac{4\pi\sigma_H}{c}\right)^4\frac{\sin^4\theta_k}{\epsilon^2}}. \qquad (4)$$

We set that for any value of the angle θ_k, there is an undamped wave (a "plus-wave": an undamped helicon), which is an analog of an ordinary helicon, which propagates *along* the strong magnetic field in an uncompensated conductor.[4] The absence of damping is not solely a consequence of the vanishing of the dissipative components of the conductivity (as discussed above); it is also a consequence of the fact that Landau damping (the direct absorption of wave energy by electrons) is possible only at $\omega > \omega_c$, since an electron which has absorbed energy must undergo a transition from a filled minigap to a vacant one.

The second wave, the "minus-wave," may be either exonentially damped (undergoing total internal reflection from the surface of the superlattice) or a propagating wave (similar to a wave in an insulator), depending on the relation between ω and $4\pi\sigma_H$.

Of particular interest are extremely low frequencies, with $\theta_k \neq 0$ (for definiteness, we set $\theta_k = \pi/2$):

$$k^2 = \frac{\omega^2}{c^2}\epsilon - \frac{1}{2}\left(\frac{4\pi\sigma_H}{c}\right)^2\frac{1}{\epsilon} \pm \frac{1}{2}\left(\frac{4\pi\sigma_H}{c}\right)^2\frac{1}{\epsilon}. \qquad (5)$$

We see that one of the waves is a totally ordinary wave $[k^2 = (\omega^2/c^2)\epsilon]$, while the other has an extremely unusual dispersion relation:

$$k^2 = \frac{\omega^2}{c^2}\epsilon - \left(\frac{4\pi\sigma_H}{c}\right)^2 \frac{1}{\epsilon} \quad . \tag{6}$$

In other words, at $\omega \ll 4\pi\sigma_H/\epsilon$ we should see a sort of "Meissner effect": A quasistatic electromagnetic field does not penetrate into the superlattice [at $H = H_s$ (!)] even in the limit $\omega \to 0$. The polarization of the first wave is no different from that of a linearly polarized wave in an insulator (in particular, we have $\mathbf{H} \perp \mathbf{E}$), while the second wave has not only an usual dispersion but also a "strange" polarization: The magnetic field of the wave vanishes in the limit $\omega \to 0$ (at a fixed value of the electric field).

The existence of a "wave" with $k \neq 0$ at $\omega = 0$,

$$k^2 = -\left(\frac{4\pi\sigma_H}{c}\right)^2 \frac{\sin^2\theta_k}{\epsilon}, \quad \theta_k \neq 0, \tag{7}$$

undoubtedly reflects the fact that the motion of the charges in the superlattices is nondissipative at $H = H_s$, which makes superlattices similar to superconductors.

The field H does not have to be equal to H_s for an observation of the properties of superlattices which we have described here. The localization of electrons leads to the existence of a finite interval ΔH in which the dissipative components of the conductivity tensor of the superlattice will be zero.[3]

The undamped helicon described here should apparently propagate in Hall dielectrics, whose discovery was reported in Ref. 6.

[1)]We are restricting the analysis to a zero temperature.
[2)]Tal'yanskiĭ[5] has studied quasistatic Hall oscillations in a sample of bounded dimensions.

[1]H. L. Störmer, J. P. Eisenstein, A. C. Gossard, W. Wiegman, and K. Baldwin, Phys. Rev. Lett. **56**, 85 (1986).
[2]V. M. Polyanovskiĭ, Fiz. Tekh. Poluprovodn. **17**, 1801 (1983) [Sov. Phys. Semicond. **17**, 1150 (1983)].
[3]V. N. Lutskiĭ, M. I. Kaganov, and A. Ya. Shik, Zh. Eksp. Teor. Fiz., 1986 (in press).
[4]O. V. Konstantinov and V. I. Perel', Zh. Eksp. Teor. Fiz. **38**, 161 (1960) [Sov. Phys. JETP **11**, 117 (1960)]; see also A. A. Abrikosov, Vvedenie v teoriyu normal'nykh metellov (Introduction to the Theory of Normal Metals), Nauka, Moscow, 1972, Ch. XI.
[5]V. I. Tal'yanskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **43**, 96 (1986) [JETP Lett. **43**, 127 (1986)].
[6]S. S. Murzin, Pis'ma Zh. Eksp. Teor. Fiz. **44**, 45 (1986) [JETP Lett **44**, 56 (1986)].

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