Absolute instability of a "trapped" packet of sound waves in an anisotropic-semiconductor plate

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It is shown that oblique waves incident on a conducting plate with strong anisotropy of the elastic and acoustoelectric properties become absolutely acoustically unstable when amplified by the electron drift.

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It is well known that in anisotropic semiconductors, such as II—VI compounds, the directions of the phase and group velocities **v** and $\mathbf{v}_g$ of the acoustic waves are in general different.

1. We show first that the orientation of a plane-parallel plate made of such a material can be chosen such that the group velocities of the waves incident (i) on its planes and those reflected (r) are parallel (Fig. 1). In this case the wave packet, after passing twice between the reflecting planes, does not move along the planes—it remains trapped in the plate. The trapping condition is satisfied if the angles between the chosen crystallographic axis C and the group velocities are connected by the relation

$$\theta_g^i(\theta^i) = \theta_g^r(\theta^r) - \pi. \qquad (1)$$

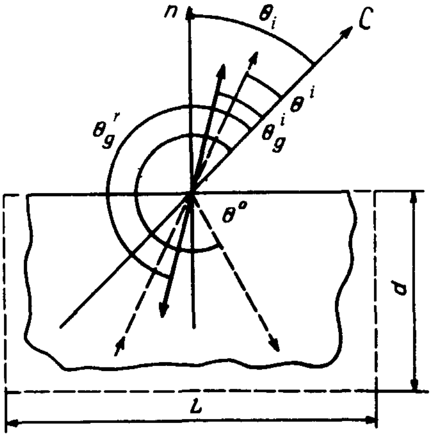

FIG. 1. Diagram of wave reflection ($v_d \parallel n$).

Here θ and θ_g are the angles between the C axis and the phase and group velocities, respectively.

From the reflection law

$$\frac{v^i(\theta^i)}{\sin(-\theta^i+\theta_n)} = \frac{v^r(\theta^r)}{\sin(\theta^r-(\theta_n+\pi))} \tag{2}$$

for angles $\theta^{i,r}$ satisfying the condition (1), we easily obtain the orientation of the C axis relative to the normal—the angle θ_n (Fig. 1). For crystals in which the directions parallel and antiparallel to the wave vector qe are equivalent, i.e., $v(\theta)=v(\theta)+\pi)$, Eq. (1) takes the form

$$\theta_g^i(\theta^i)=\theta_g^r(\theta^r),\ \tilde{\theta}^r=\theta^r-\pi. \tag{1'}$$

This equation can be solved for θ^i and θ^r graphically, by plotting θ_g against θ (Fig. 2). The solutions of (1) are respectively the points where the straight line θ_g=const intersects the plot of $\theta_g(\theta)$. Each point of the $\theta_g(\theta)$ plot is in general a solution of Eq. (1) for waves normally incident on the surface ($\theta^i=\tilde{\theta}^r=\theta_n$) and having group velocities that make an angle with the normal. For the case of oblique waves, Eq. (1′) has solutions when $\theta_g(\theta)$ is a multiply valued function. Each chosen pair of angles θ^i and $\tilde{\theta}^r$ satisfying (1′) corresponds to a

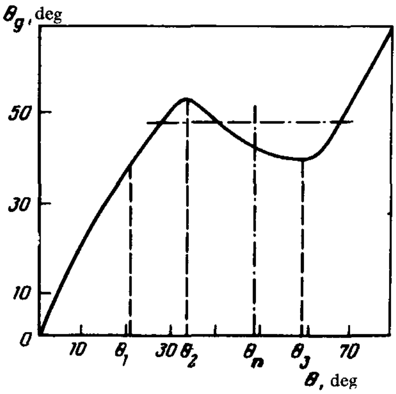

FIG. 2. Plot of $\theta_g(\theta)$ in CdSe crystal for quasi-transverse waves: $\omega=\omega_{max}$, $v_d=0$.

different crystal orientation—the angle θ_n of (2). For example, in a plate of the piezo-semiconductor CdSe, trapping of the sound packet can be observed for quasitransverse waves propagating at angles to the C_{6V} axis in the region $\theta_1 < \theta^i < \theta_3$ (Fig. 2), when $\theta_2 < \theta_n < \theta_3$.

2. If the "trapped" waves in the semiconductor are amplified by carrier drift during their "round trip" then absolute acoustic instability (AAI) is produced in the plate. This type of instability is a generalization of the particular case[2] $\theta_g \equiv \theta$, in which the trapping condition (1) is satisfied only for normally incident waves and only the anisotropy induced by the carrier drift in the electron decrement leads to AAI.

Owing to the strong dependence of θ_g on the conductivity σ, on the drift velocity $\mathbf{v}_d$, on the frequency ω, and on the electromechanical coupling constant $\eta(\theta)$,[1] the AAI of oblique waves has singularities that do not occur in the model of[2]. First, the condition (1) for the trapping of the packed is satisfied only for waves of one frequency at specified crystal orientations, at a specified incidence angle θ^i, and at fixed values of σ and $v_d = \mu U/d$. Second, since the incident and reflected waves travel at different angles to the C axis, the orientation can be chosen such as to satisfy the optimal condition for sound amplification: $\eta(\theta^i) \gg \eta(\theta^r)$ $(\theta^i, \theta^r \neq \theta_n)$, i.e., the electrons are strongly coupled with the incident wave and weakly with the reflected wave. From the expression for the growth rate on the "round trip" paths:

$$\Gamma_{\Sigma} = (\Gamma_+ + \Gamma_- - 2\Gamma_{\text{vis}})d + 2\ln R^i R^r,$$

$$\Gamma_{\pm} = \frac{\eta^2_{i,r}\ \omega_M(\mathbf{q}^{(i,r)}\ \mathbf{v}_d/\omega - 1)}{v_g^{(i,r)}\ [(\mathbf{q}^{(i,r)}\ \mathbf{v}_d/\omega - 1)^2 + \omega_M^2(1 + (q^{(i,r)}\ r_D)^2)^2/\omega^2]}, \quad (3)$$

(where $\omega_M = \sigma/\epsilon$, ϵ is the dielectric constant and r_D is the Debye radius), it is seen that the amplification of the sound ($\Gamma_\Sigma > 0$), is caused in effect by the fact that the growth rate Γ_+ of the waves traveling along the drift exceeds the losses of non-electron type (Γ_{vis}) and losses to reflection ($R^{i,r}$ are the reflection coefficients), since the electronic decrement of the sound in the return path is small, $\Gamma_- \approx 0$. Therefore AAI of oblique waves can exist in the region of high sample conductivities and high sound frequencies, for which there is no AAI of normally incident waves: $|\Gamma_-| > |\Gamma_+|$ at $\eta(\theta^i) = \eta(\theta^r)$, $(\theta^i, \widetilde{\theta}^r = \theta_n)$. It is precisely in this region that an experiment was performed aimed at distinguishing the AAI of oblique waves from the noise, since the effect in question holds an equal degree also for the amplification of acoustic noise.

3. We chose a photosensitive CdSe crystal of thickness $d = 0.2$ mm, length $L = 0.2$ mm, width $h = 0.8$ mm, carrier mobility $\mu = 380$ cm^2/V-sec, and orientation $\theta_n = 49°$ such that (see Fig. 2) the trapping condition (1), namely $\theta_g^i = \theta_g^r - \pi = \theta_n$, is satisfied for a wave incident at an angle $\theta^i = 31°$ ($\eta_i^2 = 2.7 \times 10^{-2}$) and reflected at angle $\theta^r = 67°$ ($\eta_r^2 = 5 \times 10^{-4}$) at a frequency ω close to the frequency of the maximum gain $\omega_{\max} = v/r_D$. The conductivity was chosen in the region $\sigma > 3 \times 10^{-3}\ \Omega^{-1}\text{cm}^{-1}$, where the conditions for the existence of AAI of normally incident waves are not satisfied. Just as in the case of waves traveling in the normal direction $\mathbf{q} \parallel \mathbf{n}$, the AAI of oblique waves was revealed by the change of the current (Fig. 3) due to the appearance of the acoustoelectric current I_{ae} as a result of amplification of the acoustic noise by the carrier drift.[3] We mea-

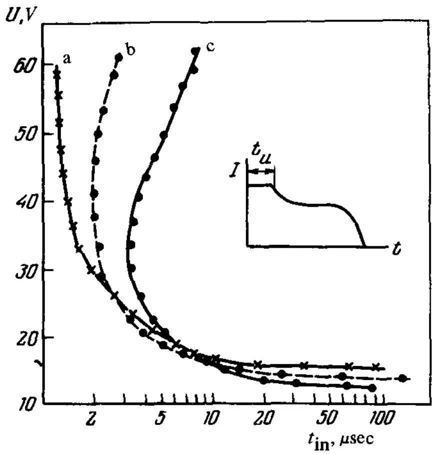

FIG. 3. Plots of t_{in} against the applied voltage V for different conductivities $\sigma \times 10^3$ $\Omega^{-1}\,cm^{-1}$: a—23, b—11, c—6.25.

sured the field dependences of the "incubation" (t_{in}) time of the deviation of the current from the Ohmic value

$$\Delta I = I_{ohm} - I(t_{in}) = I_{ae}(t_{in}).$$

This deviation was chosen such that the nonlinear effects were still weak at the corresponding sound intensity S.[4] We can therefore estimate t_{in} from the expression for $I_{ae} = hL\Gamma_+\mu S/v = hL\Gamma_+\mu S_0\Omega\exp\{\Gamma_\Sigma t_{in}/\tau\}/v$ in the single-frequency approximation of the linear theory:

$$\Gamma_\Sigma \frac{t_{in}}{\tau} - \ln\frac{S}{S_0} = 2\ln\frac{2dt_{in}}{L\tau}, \quad \Omega = \left(\frac{L\tau}{2dt_{in}}\right)^2. \tag{4}$$

S_0 is the equilibrium intensity of the noise in a unit solid angle, and τ is the wave "round trip" travel time. In (4) we took into account the contribution made to I_{ae} by those waves which did not leave the volume in which the amplification takes place during the time t_{in}, since it is precisely these waves which determine the time of current saturation. The group velocities of these waves make an angle $|\theta_g - \theta_n| < L\tau/2dt_{in}$ with the normal. As seen from Figs. 3a,b,c, at $\sigma > 3\times10^{-3}\ \Omega^{-1}\,cm^{-1}$ and low voltages, the deviation of the current from the Ohmic value I_{ohm} occurs within times $t_{in} \approx 100\ \mu$sec, this being due to the amplification of the oblique waves with $|\theta_g^i - \theta_g^r + \pi| < 2'$. The AAI set in under the conditions of the experiment at the maximum-gain frequencies, $f_{max} = 0.78$, 1, 1.48 GHz for $\sigma = (6.25,\ 11,\ \text{and}\ 23)\times10^{-3}\ \Omega^{-1}\,cm^{-1}$, respectively. The change of the incubation time with changing field and sample conductivity is determined by the dependence of Γ_Σ on the conductivity and on the applied voltage U (see (3)). Calculation of $t_i\ (\sigma, U)$ by formula (4) shows (see Fig. 3) good agreement with the experimental results.

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