Optical analog of the magnus effect

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The scattering of electromagnetic waves by a rotating conducting cylinder was considered in[1–3]. We wish to call attention here to one physical effect accompanying such a scattering, namely, when the rotating body interacts with an incident plane electromagnetic wave, the body is acted upon by a force perpendicular to the direction of the incident wave. The order of magnitude of this effect can be estimated in the following manner.

It is easy to show than an asymmetrical scattering pattern is produced when this wave is scattered by a rotating body. Let a plane electromagnetic wave of frequency ω be incident from vacuum on a rotating cylinder of infinite length. The dielectric constant of the cylinder is ϵ, its magnetic permeability is μ, and its conductivity is σ. The radius of the cylinder is a and its angular velocity is Ω. The wave vector **k** of the incident wave makes a right angle with the cylinder axis.

The problem of the angular distribution of the scattered field will be considered under the following simplifying assumptions: 1) A small difference $(\epsilon\mu - 1) \ll 1$ and a low conductivity σ. Satisfaction of these conditions makes it possible to neglect the reflection and refraction of the light by the surface of the cylinder and the absorption of the light into the interior of the cylinder. 2) The angular velocity of the cylinder is such that the corresponding linear velocities of its elements are small in comparison with the speed of light: $u \approx \Omega a \ll c$. This condition allows us to restrict ourselves to effects of first order in u/c. 3) The cylinder dimensions are much larger than the wavelength of the incident radiation: $a \ll \lambda = 2\pi c/\omega$. Under these conditions the propagation of light inside the rotating cylinder can be treated as the propagation of a plane wave.

The volume elements of the rotating cylinder move with linear velocities that differ both in magnitude and direction. Let the velocity of a volume element be **u**, and let the vector **u** make an angle θ_0 with the vector of the incident wave. The phase velocity of the light in this volume element is then[4]

$$v_{ph} = \frac{c}{\sqrt{\epsilon\mu}} + \left(1 - \frac{1}{\epsilon\mu}\right) u \cos\theta_o \ . \qquad (1)$$

The velocity u of the elements of the rotating cylinder is of the order of $a\Omega$, and the angle θ_0 ranges from 0 to 2.

It is seen from (1) there is no dragging effect at $\theta_0 = \pm\pi/2$. At $\theta_0 = 0$ or $\theta_0 = \pi$ the dragging effect is maximal, and the phase velocity of the light is of the order of $(c/\sqrt{\epsilon\mu}) + u[1 - (1/\epsilon\mu)]$ and of the order of $(c/\sqrt{\epsilon\mu}) - u[1 - (1/\epsilon\mu)]$ in that part of the cylinder where the medium and the wave move opposite to each other.

Since these phase velocities are different, the front of the wave will turn. A wave leaving the cylinder propagates at a certain angle α to the initial direction. Simple estimates show that the deflection angle α is of the order of

$$\alpha \approx \frac{2u}{c} \sqrt{\epsilon\mu}\left(1 - \frac{1}{\epsilon\mu}\right) \approx \frac{2\Omega a}{c} \sqrt{\epsilon\mu}\left(1 - \frac{1}{\epsilon\mu}\right). \qquad (2)$$

The wave is deflected from its initial direction in the direction of the cylinder rotation.

The deflection changes the electromagnetic momentum of the wave incident on the cylinder. Let us estimate this change. If the electric field intensity in the incident wave is E_0, then the radiation momentum flux incident on a unit cylinder length per unit time is $p_0 = (a/4\pi c)E_0^2$. As a result of scattering through the small angle α, the momentum flux p_0 changes by an amount $\Delta p_0 = \alpha p_0$. The direction of the vector $\Delta \mathbf{p}_0$ coincides with the direction of the vector $\mathbf{\Omega} \times \mathbf{k}$, where Ω is the vector of the angular velocity of the cylinder and $\mathbf{k}$ is the incident-wave vector. By virtue of the momentum conservation law, a unit length of the cylinder is acted upon by a deflecting force numerically equal to Δp_0 but of opposite direction. Taking into account the value of the deflection angle (2), we obtain an estimate for the deflecting force acting on a unit cylinder length:

$$\mathbf{F} = -[\vec{\Omega} \times \mathbf{k}] \frac{a^2 E_o^2}{2\pi\omega c} \sqrt{\epsilon\mu}\left(1 - \frac{1}{\epsilon\mu}\right) = -[\vec{\Omega} \times \mathbf{k}] \frac{4a^2 I_o}{\omega c^2} \sqrt{\epsilon\mu}\left(1 - \frac{1}{\epsilon\mu}\right), \qquad (3)$$

where $I_0 = (c/8\pi) E_0^2$ is the intensity of the incident radiation. This effect can also be calculated with the aid of the Kirchoff approximation of diffraction theory. This approximation takes into account the difference between the phase shift for a wave passing through different sections of the rotating cylinder. Knowledge of the phase of the wave "at the exit" from a dielectric cylinder (on the surface) makes it possible to calculate the scattered field.

To determine the phase "at the exit" we take into account the cylinder rotation in a simplified manner, representing the cylinder by two moving dielectric layers of thickness a. The velocity of one of the layers is βc and coincides with the direction of the incident wave, while the velocity of the second layer is equal but opposite.

In this approximation we obtain for the scattered field E_p the expression

$$E_p = E_o \sqrt{\frac{k}{2\pi i R}}\, e^{ikR}\left\{2\pi\delta(k\sin\theta) - 2\,\frac{\sin(ka\sin\theta)}{k\sin\theta} + 4\,\frac{\sin\left(\frac{1}{2}ka\sin\theta\right)}{k\sin\theta}\right.$$

$$\left.\times\cos\left[\frac{1}{2}ka\sin\theta - 2ka\beta(\epsilon\mu - 1) - \frac{4\pi i\sigma}{c}\beta\mu a\right]\exp\left[-2ika(1-\sqrt{\epsilon\mu}) - \frac{4\pi\sigma}{c}\sqrt{\frac{\mu}{\epsilon}}\,a\right]\right\}. \quad (4)$$

where θ is the scattering angle (i.e., the angle between the direction of the incident wave and the radius vector **k** of the observation point); $u=\beta c=\Omega a$ is the displacement velocity on the surface of the rotating cylinder; $k=\omega/c$ is the magnitude of the wave vector. The first term of (4), which is proportional to a delta function, yields the undeflected wave. If $\epsilon=\mu=1$ and $\sigma=0$ (there is no cylinder), this is the only remaining term. The second term describes the diffraction of a wave by an opaque strip of width $2a$. As $\sigma\to\infty$, the entire expression (4) reduces to the sum of the first two terms. The third and last term in (4) takes into account the influence of the rotation of the cylinder. This term decreases exponentially with increasing cylinder conductivity σ. As $\sigma\to\infty$, we can easily estimate the angle through which the wave is deflected as a result of the scattering. Expression (4) has a maximum with respect to the angle θ at $\theta=0$ (due to the term with the δ function) and at

$$\theta_{max} = \frac{4\Omega a}{c}(\epsilon\mu - 1). \quad (5)$$

Under our assumptions this expression agrees satisfactorily with (2).

We note that the scattering-pattern asymmetry and the ensuing deflecting force occur not only when the body rotates, but also when the optical thickness of the scattering body varies along the front of the incident wave.

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[1]Ya.B. Zel'dovich, Pis'ma Zh. Eksp. Teor. Fiz. **14**, 270 (1971) [JETP Lett. **14**, 180 (1971)].
[2]Ya.B. Zel'dovich, Zh. Eksp. Teor. Fiz. **62**, 2076 (1972) [Sov. Phys. JETP **35**, 1085 (1972)].
[3]B.M. Bolotovskiĭ and S.N. Stolyarov, Zh. Eksp. Teor. Fiz. **71**, 1003 (1976) [Sov. Phys. JETP **44**, 525 (1976)].
[4]I.E. Tamm, Osnovy teorii élektrichestva (Principles of the Theory of Electricity), Nauka, 1976.