Instability in a theory with strong coupling of vacuum reggeons

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It is shown that a theory with strong pomeron coupling is contradictory because it includes an instability with respect to the parameter that determines the dependence of the inclusive particle-production vertices on the pomeron momentum.

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In the infrared asymptotic representation, the Green's function of a vacuum reggeon is of "scale-invariant" form[1,2] corresponding to strong coupling of the pomerons. We consider below this solution in light of inelastic processes.

It is convenient to investigate many-particle processes by introducing generating functions, for which a sufficiently simple reggeon diagram technique can be constructed.[3,4] Within the framework of this diagram technique, one introduces propagators that correspond to a "cut" pomeron[3,4] (the rules for "cutting" the reggeon pomerons are indicated in[5]):

$$D^{-1}(\omega,\ k^2;\ z) = \omega + \omega_R'(z)k^2 + m_R(z), \tag{1}$$

where z is proportional to the number of particles per rapidity unit. The generating functions are so normalized that at $z=1$ they should coincide with the imaginary part of the corresponding amplitude of the elastic scattering.[4] This normalization condition should be preserved during any stage of the calculation of the generating function. Then the "slope" $\omega_R'(z)$ of the cut pomeron should satisfy the condition

$$\omega_R'(r)\Big|_{z=1} = \alpha_R' \ , \tag{2}$$

where α_R' is the renormalized slope of the pomeron trajectory. The assumption $\omega_R'(z)\equiv\alpha_R'$ was chosen in[6], but we shall assume here that $\omega_R'(z)$ is a nontrivial function of z, a natural assumption, for example, in the multiperipheral model. The opposite assumption means, inasmuch as by definition[4]

$$\omega_R'(z) \equiv \alpha_R' + \sum_{\nu=1}^{\infty} (z-1)^{\nu} \frac{d}{dk^2}\Psi_\nu(k^2)\Big|_{k^2=0}, \tag{3}$$

that the conditions

$$\frac{d}{dk^2}\Psi_\nu(k^2)\Big|_{k^2=0} = 0 \tag{4}$$

must be satisfied for all $\nu = 1,2,\ldots$; here Ψ_ν are the vertices of the inclusive production of particles in Mueller-Kancheli diagrams (see[7]). The solution suggested in[1,2] presupposes that the "intersection" $m_R(z)$ must satisfy the following normalization condition

$$m_R(z)\Big|_{z=1} = 1 - \alpha_R(0) = 0. \tag{5}$$

The subscript R will henceforth be used to label renormalized parameters.

A detailed examination of the manner in which the three-pomeron vertices J are renormalized allows us to state that at $z \neq 1$ they depend on the method by which they are cut[4] (we assume here that the bare vertices do not depend on the cutting method, i.e., $J_i \equiv J_0$). However, in view of the aforementioned normalization conditions, we must put

$$J_{i,R}(\omega, z)\Big|_{z=1} = J_{0,R}(\omega), \tag{6}$$

where i and 0 number the cut pomerons going into the vertex. We consider a solution in which $J_{0,R}$ has a finite infrared limit λ.[1,2] From this and from (6) it follows that

$$J_{i,R} \equiv J_{0,R} + \sum_{\nu=1}^{\infty} (z-1)^{\nu} r_{i,R}^{\nu} = \lambda + \Phi_i(\omega, z), \tag{7}$$

where Φ_i are certain non-universal functions that vanish in the infrared limit. The latter follows from the fact that $r_{i,R}^{\nu}$ are "inessential parameters"[8] in the limit as $\omega \to 0$ and in the space n in which the considered theory is renormalizable.

Using the renormalization-group approach formulaed in[9], we can obtain equations for $J_{i,R}$ at arbitrary z:

$$\omega \frac{dJ_{i,R}}{d\omega} = P_i(J_{0,R}, J_{1,R}, \ldots, J_{3,R}; \kappa), \tag{8}$$

where $\kappa_R = \alpha'_R/\omega'_R$. As a result of the conditions (2) and (6), the system of equations (8) should degenerate at $z=1$ into one equation for $J_{0,R}$, i.e., we must put

$$P_i(J_{0,R}, \ldots, J_{3,R}; \kappa_R)\Big|_{z=1} = P_0(J_{0,R}) \underset{\omega \to 0}{\to} P_0(\lambda) = 0. \tag{9}$$

However, if we substitute (7) in (8) and use the fact that the Φ_i vanish at $\omega = 0$, we obtain in the limit as $\omega \to 0$:

$$P_i(\lambda; \kappa) = 0, \qquad \kappa \equiv \kappa(z) \equiv \kappa_R(\omega, z)\Big|_{\omega=0}. \tag{10}$$

If we compare this condition with (9), then it becomes more natural to assume that, independently of the value of z,

$$\kappa_R(\omega, z)\Big|_{\omega=0} \equiv \kappa(z) = 1. \tag{11}$$

The necessity of the condition (11) can be proved by calculating the explicit form of $P_i(\lambda; \kappa)$. Since we are interested only in the self-consistency of the theory, we can consider it in a space in which $0 < \epsilon = 4 - n \ll 1$. By using the analysis proposed in[10] we can prove that the ϵ expansion is correct in the considered theory. We can then confine ourselves to the single-loop approximation in the calculation of P_i, and thus obtain the explicit form of Eqs. (10), which must be solved relative to κ:

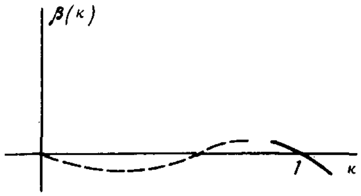

FIG. 1.

$$P_1(\lambda;\kappa) = \frac{\epsilon}{4}\lambda\left\{\frac{\kappa^2}{3}\left(1+\frac{8}{(1+\kappa)^2}\right)-1\right\}=0, \tag{12a}$$

$$P_2(\lambda;\kappa) = \frac{\epsilon}{4}\lambda\left\{\frac{2}{3}\left(2\frac{1+3\kappa}{1+\kappa}-\frac{6\kappa}{(1+\kappa)^2}-\kappa\right)\kappa-1\right\}=0, \tag{12b}$$

$$P_3(\lambda;\kappa) = \frac{\epsilon}{4}\lambda\left\{\frac{1}{3}\left(2-5\kappa^2+\frac{32\kappa^2}{1+\kappa}-\frac{40\kappa^2}{(1+\kappa)^2}\right)-1\right\}=0\;. \tag{12c}$$

These equations demonstrate clearly the uniqueness of condition (11).

In the considered single-loop approximation, the renormalization-group equation for κ_R is

$$\omega\frac{d\kappa_R}{d\omega} = -\frac{\epsilon}{24}\kappa_R\left(3-\kappa_R+2\kappa_R^2-\frac{32\kappa_R^2}{(1+\kappa_R)^3}\right)\equiv\frac{\epsilon}{24}\beta(\kappa_R). \tag{13}$$

It is easy to see that $\beta(\kappa_R)$ has a zero at $\kappa_R=1$, but the derivative of $\beta(\kappa_R)$ is negative at this point (corresponding to $z=1$), see Fig. 1.

The strong-coupling solution will be stable to variations of the parameters if the condition (11) is satisfied, i.e., only if the point $z=1$ is a stable point of the theory: the changes of κ_R due to the deviation of z from unity should "fade out," according to (11) in the asymptotic limit $\omega\to 0$. However, as follows from Fig. 2, which shows that solutions of (13) in the vicinity of the point $z=1$, the point $z=1$ is not a stable point of the theory, thus indicating that the solution with strong coupling of the vacuum reggeons is contradictory. The contradiction manifests itself formally in the fact that Eqs. (8) and (13) have no compatible solutions.

In our next paper we propose to present in complete form all the proofs schematically outlined in the present paper, and also discuss the physical

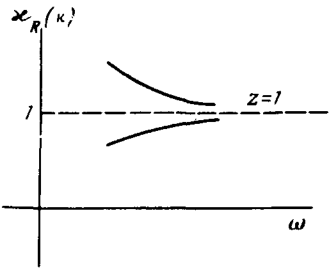

FIG. 2.

meaning of the instability of the solution with strong coupling of pomerons. Our result means, using the terminology of statistical physics, that the phase transition that should take place in a "reggeon gas" by virtue of condition (5) is impossible.

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