Width of compression-wave front in nuclear matter

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The behavior of the specific volume and of other macroscopic quantities in the transition layer of a shock wave in nuclear matter is considered in the hydrodynamic approximation with viscosity taken into account. It is shown that the transition layer is strongly elongated in the direction of the incident flux.

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A number of recent papers[1,2] deal with the propagation of shock waves in nuclei. The first groups of papers[1] is based on the hydrodynamic approach, whereas the second[2] uses various modifications of the cascade model. The density profiles of the medium, calculated in the cascade model, turn out to differ from the discontinuous solutions in hydrodynamics, so that the applicability of the latter becomes doubtful. Of course, the hydrodynamic approximation is satisfied for real nuclei only in order of magnitude, but the ideal-liquid approximation, which leads to discontinuous solutions, calls for satisfaction of even more stringent criteria.

We consider in this paper the w ock-wave front in hydrodynamics, with allowance for viscosity and thermal conductivity of the medium. A substantial factor here is that the temperature of the liquid tends to zero with increasing distance from the front in the direction of the incident flow. It is known[3] that the viscosity coefficient increases in this case in proportion to T^{-2}, and the thermal conductivity coefficient in proportion to T^{-1}. We see therefore that the medium is far from ideal in this vicinity, and allowance for viscosity is particularly important. We consider for simplicity the case of low-intensity waves, in analogy with the procedure used in[4].

The energy and momentum flux-density conservation laws lead to the following equations (in the stationary-flow system):

$$P + j^2 V - j\left(\frac{4}{3}\eta + \zeta\right)\frac{dV}{dx} = j V_0 , \tag{1}$$

$$w + \frac{j^2 V^2}{2} - j\left(\frac{4}{3}\eta + \zeta\right) V \frac{dV}{dx} = w_0 + \frac{j^2 V_0^2}{2} \tag{2}$$

(P is the pressure, V and w are the volume and thermal function per unit mass, j is the mass flux; the subscript "0" labels quantities in the incoming stream). The internal energy per unit mass is written in the form

$$\epsilon = \epsilon_k + \epsilon_T, \tag{3}$$

where ϵ_k is the energy of the cold matter, equal to

$$\epsilon_k = \epsilon_0 + \frac{k}{2m}\frac{(V_0 - V)^2}{V^2} \tag{4}$$

(ϵ_0 is the equilibrium-state energy, k is the compressibility, and m is the nucleon mass). For small compressions T^2 is proportional to the cube of the change of the volume, therefore the quasiparticles that appear as a result of heating can be regarded as an ideal gas. The temperature-dependent parts of the internal energy (ϵ_T) and of the pressure (P_T) are then connected with the temperature (T) by the simple relations

$$\epsilon_T = \frac{2}{3} P_T V = \frac{1}{3}\frac{p_F V T^2}{\hbar^3} . \tag{5}$$

(p_F is the Fermi momentum). Relations (4) and (5) enable us to find ϵ as a function of the pressure and of the specific volume. Eliminating the derivative from (1) and (2), we obtain the connection between the volume and the pressure in the transition layer. Equation (1) for low-intensity waves now takes the form

$$\left(\frac{4}{3}\eta + \zeta\right)\frac{dV}{dx} = -\frac{25}{12} j \frac{(V_0 - V)(V - V_1)}{V_0^3} \tag{6}$$

(V_1 is the specific volume in the outgoing stream). Using the expressions obtained by Abrikosov and Khalatnikov[3] for the viscosity and relation (5) for T^2, we arrive at a simple equation for V. Its solution is

$$V - \frac{V_0 + V_1}{2} = \frac{V_0 - V_1}{2} F\left(\frac{x}{\delta}\right), \tag{7}$$

where the width δ of the transition layer is equal to

$$\delta = l_0\left(\frac{2V_0}{V_0 - V_1}\right)^4 \tag{8}$$

(l_0 is the nucleon mean free path, approximately 1 F). For large positive x (outgoing stream), Eq. (7) takes the form

$$V - V_1 \sim (V_0 - V_1)e^{-x/\delta}. \tag{7'}$$

For large negative x (incoming stream) we have

$$V_0 - V \sim (V_0 - V_1)\sqrt{\frac{\delta}{|x|}}. \tag{7''}$$

It is easy to verify that this square-root dependence is determined only by the behavior of the viscosity coefficient, and is therefore valid also for high-intensity waves.

The transition layer is consequently always strongly elongated in the direction of the incoming stream.

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