Possibility of reconstructing the pattern of conduction-electron scattering by the sample boundary from the experimental data

O. V. Kirichenko, V. G. Peschanskiĭ, and S. N. Savel'eva

A. F. Ioffe Physicotechnical Institute, USSR Academy of Sciences and Physicotechnical Institute of Low Temperatures, Ukrainian Academy of Sciences

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It is shown that investigations of the kinetic phenomena in thin metallic conductors, particularly the oscillatory dependence of the kinetic coefficient on the strong magnetic field H and on the sample thickness d, determines uniquely certain parameters of the pattern of carrier scattering by the conductor boundary.

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In the theory of kinetic phenomena, the scattering of electrons by the surface of a conductor is taken into account with the aid of the boundary condition for the conduction-electron distribution function

$$f(\mathbf{p}, \mathbf{r}_s) = \int f(\mathbf{p}', \mathbf{r}_s) w(\mathbf{p}, \mathbf{p}', \mathbf{r}_s) G(d\mathbf{p}'), \tag{1}$$

where $w(\mathbf{p}, \mathbf{p}', \mathbf{r}_s)$ is the probability that the electron incident on the sample surface at the point $\mathbf{r}_s$ with momentum $\mathbf{p}'$ will have momentum $\mathbf{p}$ after reflection. The quantity G is determined from the normalization condition.

It has been customary, however, to use in the calculation of the kinetic characteristics of conductors not the condition (1) but the simpler condition describing the reflection of the electrons with the aid of the phenomenological specularity parameter introduced by Fuchs.[1] This description is approximate and not always suitable. The boundary condition obtained in a number of theoretical papers from considerations of random roughnesses and defects of the surface[2] differs substantially from Fuchs's condition, and this raises the question of the extent to which it is important to know the explicit form of the function w for the kinetics of thin conductors.

Assuming that w is an arbitrary but specified function of its arguments, we have calculated the kinetic coefficients of metallic conductors with thicknesses d much smaller than the carrier mean free path l. The oscillatory dependence of the resistivity, of the thermal resistance, and of the thermoelectric power on the value of the strong magnetic field (electron-trajectory curvature radius $r \ll d$), which was predicted by Sondheimer[3] has turned out to be extremely sensitive to the shape of the scattering pattern w.

We present by way of example the oscillatory dependence, on H and d, of the resistivity of a thin plate in a magnetic field perpendicular to its surfaces, which are symmetry planes of the crystal.

The electron distribution function takes in this case the very simple form

$$f(\mathbf{p}, \mathbf{r}) = f_0(\epsilon) - \Psi \frac{\partial f_0}{\partial \epsilon};$$

$$\Psi = \int_{\lambda_1}^{t} g_1(t', t, p_z)dt' + \int w(\mathbf{p}_1, \mathbf{p}'; \mathbf{r}_1) G(dp') \int_{\lambda_2}^{\lambda_1} g_2(t', t, p_z')dt'$$

$$+ \int w(\mathbf{p}_1, \mathbf{p}'; \mathbf{r}_1) G(dp') \int w(\mathbf{p}_2, \mathbf{p}''; \mathbf{r}_2) G(dp'') \int_{\lambda_3}^{\lambda_2} g_3(t', t, p_z'')dt' \quad ...;$$

$$g_i(t', t, p_z) = \mathbf{v}(t')\mathbf{E}(\mathbf{r}_i - \mathbf{r}(\lambda_i) + \mathbf{r}(t')) \exp \frac{t' - t}{\tau}. \qquad (2)$$

The function Ψ has the meaning of the energy acquired by the charge in the electric field $\mathbf{E}$ and averaged over all possible paths of the charge in the thin sample; $f_0(\epsilon)$ is the electron Fermi distribution function; e, $\mathbf{v}$, and p_z are the electron charge, velocity, and momentum projection on the magnetic-field direction; $\tau = l/v$; t is the time of motion of the electron at the instant λ_i of its reflection by the sample boundary at the point $\mathbf{r}_i$; the λ_i are the roots of the equations

$$\int_{\lambda_1}^{t} \mathbf{v}(t')dt' \equiv \mathbf{r}(t) - \mathbf{r}(\lambda_1) = \mathbf{r} - \mathbf{r}_1; \quad \mathbf{r}(\lambda_i) - \mathbf{r}(\lambda_{i-1}) = \mathbf{r}_i - \mathbf{r}_{i-1}; \qquad i > 1.$$

Using formula (2), we easily calculate the density of the electric current and obtain the resistivity of the conductor. If the electron reflection is not too close to specular and the reflection-angle interval Δ in which the scattering pattern changes substantially is large in comparison with r/d, then the contribution to the oscillatory dependence of ρ on H and d from the electrons near the turning point of the Fermi surface is determined mainly by the third term in the expression for Ψ, and ρ^{osc} takes the following form:

$$\frac{\rho^{osc}}{\rho} = \frac{r}{d} e^{-d/l} \frac{W_1 \sin d/r}{d/l + W_0}; \quad \Delta >> r/d, \qquad (3)$$

where

$$W_0 = \left\langle v_z v_\perp \left[v_\perp - \frac{1}{p} \int_0^p W(p_z, -p_z') v_\perp(p_z') dp_z' \right] \right\rangle;$$

$$W_1 = \left\langle v_z v_\perp W(p_z, -p) \frac{1}{p} \int_0^p W(-p, p_z') v_\perp(p_z') dp_z' \right\rangle;$$

$$W(p_z, p_z') = \frac{1}{2\pi} \int_0^{2\pi} d\phi \, e^{i\phi} w(p_z, p_z', \phi).$$

The angle brackets denote integration with corresponding normalization over all the states of the reflected electrons; $v_\perp = \sqrt{v^2 - v_z^2}$, while $w(p_z, p_z', \phi)$ is the probability of elastic scattering of the electron by the sample boundary, such that p_z' changes by p_z and the phase on the electron trajectory acquires in a magnetic field an increment ϕ. The function $w(p_z, p_z', \phi)$ does not depend on $\mathbf{r}_s$, inasmuch as the expression for the electric-current density is averaged over the entire volume of the conductor, while w is an averaged characteristic of the surface.

If the width Δ of the scattering pattern is small in comparison with r/d, then the specularity-parameter approximation[4] is valid

$$\frac{\rho^{\text{osc}}}{\rho} = (r/d)^2 e^{-d/l}(1-q)^2 \sin d/r\ ; \quad \Delta \lesssim r/d\ . \tag{4}$$

Thus, with increasing magnetic field, the quadratic dependence of the amplitude of the Sondheimer oscillations on $1/H$ gives way, in magnetic fields for which $r/d \approx \Delta$ to a linear dependence.

If all the directions of the reflected-electron velocities are equally probable, i.e., if w does not depend on ϕ, then $W(p_z, p_z') \equiv 0$ and it is necessary to retain the next term of the expansion of ρ^{osc}/ρ in powers of r/d; the form of this expansion coincides with formula (4) if we put $q = 0$. However, if even a weak correlation is preserved between the reflected electrons and those incident on the sample boundary, the oscillatory dependence of the resistivity on H and d is described by formula (3). The specularity-parameter approximation turns out to be valid in a very narrow region, when $q < r/d$ or when $(1-q) < r/d$. Only in the special case $w(p_z, p_z', \phi) = w(\phi)\delta(p_z + p_z')$ do the calculations of ρ^{osc} with the use of the boundary condition in the integral form (1) and in the specularity-parameter approximation yield the same result.

Thus, the oscillatory dependence on H and d is determined not only by the sharpness but also by the form of the scattering pattern. The formulas presented above for ρ^{osc} are valid also in a magnetic field making an angle with the normal, if d is replaced by $d/\cos\alpha$. An investigation of $\rho^{\text{osc}}(H)$ at different α makes it possible to determine the dependence of the width Δ of the pattern on the angle of incidence of the electrons on the sample boundary. In the calculation of the thermal conductivity κ and the thermoelectric power in thin plates, an important role is played also by the boundary condition for the heat flux. An investigation of heat transport in a plate with thermally insulated surfaces yields in fact the same information on the scattering pattern as an experimental investigation of ρ^{osc}. By investigating the oscillatory dependence of κ on H and d at a nonzero heat flow through the sample surfaces we can obtain additional information, connected mainly with the inelastic scattering of the carriers by the sample boundary, concerning the function w.

[1]K. Fuchs, Proc. Cambr. Phil. Soc. **34**, 100 (1938).
[2]F.G. Bass, V.D. Freĭlikher, and I.M. Fuks, Pis'ma Zh. Eksp. Teor. Fiz. **7**, 485 (1968) [JETP Lett. **7**, 373 (1968)]; A.V. Chaplik and M.V. Éntin, Zh. Eksp. Teor. Fiz. **55**, 990 (1968) [Sov. Phys. JETP **28**, 514 (1968)]; E.M. Baskin and M.V. Éntin, Zh. Eksp. Teor. Fiz. **57**, 460 (1969) [Sov. Phys. JETP **30**, 252 (1970)]; L.A. Fal'kovskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **11**, 181 (1970) [JETP Lett. **11**, 114 (1970)]; Zh. Eksp. Teor. Fiz. **58**, 1830 (1970) [Sov. Phys. JETP **31**, 981 (1970)]; V.I. Okulov and V.V. Ustinov, Zh. Eksp. Teor. Fiz. **67**, 1176 (1974) [Sov. Phys. JETP **40**, 584 (1975)].
[3]E.H. Sondheimer, Phys. Rev. **80**, 401 (1950).
[4]O.V. Kirichenko, V.G. Peschanskiĭ, and S.N. Savel'eva, Zh. Eksp. Teor. Fiz. **67**, 1451 (1974) [Sov. Phys. JETP **40**, 722 (1975)].