Calculation of the Weizsacker mass-formula parameters on the basis of the consistency conditions

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The consistency conditions imposed by spontaneous violation of translational invariance on the quasiparticle density ρ, on the local quasiparticle amplitude F, and on the self-consistent potential U of the nucleus are used to calculate the coefficients of the Weizsacker formula and other characteristics of nuclear matter.

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Until recently the theory of finite Fermi systems[1] dealt mainly with the solutions of problems connected with small perturbations of the ground state of the system, due either to an external field or to the addition of particles. The rigorous conditions obtained in[2–4] for the consistency of the quasiparticle density ρ, the local quasiparticle interaction amplitude F, and the self-consistent potential U of the nucleus, which are the consequence of spontaneous violation of the translational invariance, make it possible to enlarge the class of solvable problems and to raise the question of the calculation of such global characteristics as the density $\rho(r)$, the potential $U(r)$, and the parameters of the Weizsäcker mass formula

$$E_0(A) = \mu_\infty A + \gamma A^{2/3} + \beta(N - Z)^2/A + E_{\text{Coul}}. \quad (1)$$

Different methods for the reduction of the experimental data on the nuclear masses[5,6] yield several sets of the parameters μ_∞, γ, and β (see Table I). The coefficients of formula (1) are frequently calculated on the basis of the variational Hartree-Fock method with density-dependent effective interaction (with Skyrme forces).[7–9] This method, however, is at as yet not rigorous enough

We propose to calculate in this article the Weizäcker-formula parameters by using the aforementioned consistency conditions. It is well known that the

TABLE I. Characteristic of nuclear matter.

	Reduction of the experiments of [5]	[6]	Theory: Hartree-Fock calculation[9]	Theory: Present calculation
μ_∞, MeV	− 15.25	− 15.98	− 16.0	− 14.9
γ, MeV	17.07	20.76	19.8	19
β, MeV	33.16	36.5	33.8	30.3
r_0, F	1.22	1.175	1.15	1.17

quasiparticle effective mass in the nucleus is close to the vacuum value. This indicates that the velocity terms of the amplitude F are small, so that only the zeroth harmonic need be retained in this formula in first-order approximation. The consistency conditions then takes the form[3]

$$\frac{\partial U}{\partial \mathbf{r}} = \int F(\mathbf{r}, \mathbf{r}'; \rho) \frac{\partial \rho}{\partial \mathbf{r}'} d\mathbf{r}'. \tag{2}$$

The density ρ of the quasiparticles is easily obtained if $U(r)$ is given, namely $\rho(r) = \Sigma \lambda n_\lambda |\phi_\lambda|^2$, where n_λ are the occupation numbers and ϕ_λ are the wave functions of the quasiparticles. Therefore (2) is an integro-differential equation, so that $U(r)$ and $\rho(r)$ can be found if F is given. Solving Eq. (2) in a large system ($A^{1/3} \gg 1$), where the shell effects are small, we can obtain the characteristics of the nuclear matter.

We used in the calculations a Gaussian interaction $F(\mathbf{r}, \mathbf{r}'; \rho)$, which depends linearly on the density and describes satisfactorily the properties of the low-lying collective states of the nuclei[10,11]:

$$F(\mathbf{r}, \mathbf{r}', \rho) = C_0 \exp\left[\frac{(\mathbf{r} - \mathbf{r}')^2}{2r_G^2}\right]\left\{\hat{f}^{ex} + (\hat{f}^{in} - \hat{f}^{ex}) \frac{\sqrt{\rho(r)\rho(r')}}{\rho_0}\right\}, \tag{3}$$

where $C_0 = 360$ MeV-F^3, $r_G = 0.9$ F, $\hat{f} = f + f'\vec{\tau}_1\vec{\tau}_2$, $f^{in} - 0.2$, $f^{ex} = -2$, $f'^{in} = f'^{ex} = 0.8$, and $\rho_0 = 0.145$ F^{-3} is the normal nuclear density. To check how well it describes the single-particle properties of real nuclei, we have solved Eq. (2) for the ^{40}Ca nucleus by iteration. The bare potential was the single-particle potential from[12]. The iteration procedure converges rapidly, and after 5 iterations $U(r)$ changes by not more than 5 keV. The calculated energies of the neutron single-particle levels near the Fermi surface are equal to $\epsilon_{1d_{3/2}} = -13.4(-15.6)$, $\epsilon_{1f_{7/2}} = -6.5(-8.2)$, $\epsilon_{2p_{3/2}} = -3.7(-6.3)$ (the parentheses contain the experimental values). All the quantities are given in MeV. The self-consistent potential $U^n(r)$ calculated by us for neutrons is compared in the figure with the shell-model potential,[12] which accounts well for the experimental levels. It is seen that to obtain agreement with experiment it is necessary to make the potential approximately 2 MeV deeper. For comparison we note that Hartree-Fock calculations of single-particle levels[9] yield for ^{40}Ca values that differ from experiment by approximately 7 MeV. Equation (2) with the nuclear interaction (3) was then solved in similar fashion for a "large nucleus" (mass number $A = 7000$, $N = Z = A/2$, the Coulomb interaction was disregarded). As the bare values we took the density distributions corresponding to different radii $R = r_0^{(0)} A^{1/3}$, where $r_0^{(0)}$ was varied from 1 to 1.3 F. In all the cases, the iteration procedure converged, yielding $r_0 = 1.17$ F, $\mu = -14.3$ MeV, and $U_0 = -49.3$ MeV (r_0 is determined from the condition $R = r_0 A^{1/3}$, where R is the half-density-value radius, μ is the energy of the last filled level, U_0 is the well depth averaged over the internal region; the scale of the fluctuations of U_0 inside the nucleus is ~ 0.5 MeV). The value $\epsilon_F = p_F^2/2m = (9\pi)^{2/3}/8r_0^2 m = 35.1$ MeV calculated in this manner agrees with good accuracy with $\epsilon_F = \mu - U_0 = 35.0$ MeV. Substituting this value of ϵ_F in the Migdal formula[1] $\beta = \frac{1}{3}\epsilon_F(1 + 2f')$ we obtain $\beta = 30.3$ MeV.

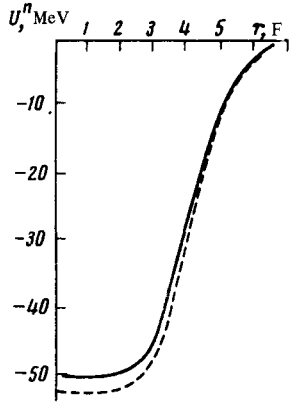

FIG. 1. Self-consistent potential of the neutrons in ^{40}Ca in the shell model[12] (dashed line) and calculated on the basis of the self-consistency condition (solid line).

The surface energy $\gamma = 4\pi r_0^2 \sigma$ was calculated on the basis of the formula obtained in[13] for the surface-tension coefficient, where it is also important to use the consistency condition (2). The value of γ depends little on the values of all the parameters of the amplitude F, with the exception of the Gaussian radius $r_G = 0.9$ we obtain $\gamma = 19$ MeV. Using these results, we can obtain μ_∞ of formula (1): $\mu(A) = \mu_\infty + \frac{2}{3}\gamma A^{-1/3}$, whence $\mu_\infty = -14.9$ MeV. We note that in our calculation there was no need for fit parameters, whereas in[7-9] the parameters of the Skyrme forces were fitted for the most part to the characteristics of the nuclear matter only, and account poorly for the single-particle spectra of nuclei.

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