

# Recombination of hydrogen in a quasi-stationary thermonuclear plasma

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We describe a calculation procedure and present data on the rate of radiative recombination of hydrogen in the plasma of quasistationary thermonuclear installations (such as tokamaks). It is shown that in modern tokamaks the concentration of the hydrogen atoms can be determined in a number of cases by radiative recombination.

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The hydrogen atoms that are present in relatively small amounts in the plasma of quasistationary thermonuclear installations (such as tokamaks) cause an appreciable cooling of the ions. The main mechanism whereby the atoms appear in the hot plasma is assumed to be their penetration into the interior of the plasma from the walls as a result of cascade charge exchange with the hydrogen ions. The atom concentration in modern tokamaks, determined experimentally and by calculation,<sup>[1]</sup> is  $n_a(0) = 10^6 - 10^7 \text{ cm}^{-3}$  at the center of the plasma and  $n_a(1) \sim 10^{10} \text{ cm}^{-3}$  at its boundary.

Under conditions of a sufficiently dense plasma with concentrations  $n \gtrsim 10^{14} \text{ cm}^{-3}$ , it is necessary to take into account one more mechanism that causes the appearance of atoms in modern tokamaks, namely radiative recombination. However there are no experimental or calculated data on the recombination cross sections at the electron temperature  $T_e \gtrsim 1 \text{ keV}$  typical for tokamaks. We describe here a procedure for calculating the recombination rates in a plasma with  $T_e = 1.5 - 10^5 \text{ eV}$ , and present the results. Estimates are made of the concentration  $n_a$  of the atoms produced as a result of this process, and these values are compared with the concentrations  $n_a$  of the atoms that penetrate into the plasma from the walls.

The following expression is given in<sup>[2]</sup> for the recombination rate when electrons are captured on all levels of the hydrogen ions:

$$\langle \sigma_p v_e \rangle = 1.27 \times 10^{-13} \frac{(I/T_e)^{3/2}}{I/T_e + 0.59} \quad (\text{cm}^3/\text{sec}) \quad (1)$$

Here  $v_e$  is the velocity of the electrons in the plasma, the angle brackets denote averaging over a Maxwellian distribution, and  $I$  is the ionization potential of the hydrogen atom ( $I = 13.6 \text{ eV}$ ). The region of applicability of formula (1) was restricted in<sup>[2]</sup> by the condition  $I/T_e > 1/32$ , i. e., to relatively low temperatures  $T_e \lesssim 400 \text{ eV}$ . Typical values for modern and next-generation tokamaks are  $I/T_e \sim 10^{-2} - 10^{-3}$ . We present below our calculation of the recombination rates for a wide range of temperatures (from  $1.5$  to  $10^5 \text{ eV}$ ).

It is known that the recombination cross section for electron capture on the 1s level can be calculated exactly within the framework of nonrelativistic quantum mechanics:

$$\sigma_r(1s) = \frac{2^8 \pi^2 \hbar e^2}{3 m^2 c} \frac{\alpha^6}{(\alpha^2 + 1)^2} \frac{e^{-4\alpha \arctan u}}{1 - e^{-2\pi\alpha}}; \quad \alpha = \frac{Ze^2}{\hbar v}. \quad (2)$$

Here  $m$  is the electron mass,  $\hbar$  is Planck's constant,  $c$  is the speed of light,  $e$  is the charge of the electron,  $v$  is the relative velocity of the atom and electron, and  $Z$  is the charge of the ion nucleus. In accordance with,<sup>[3]</sup> we can connect the total recombination cross section  $\sigma_r$  with the cross section  $\sigma_r(1s)$  for recombination on 1s level:

$$\sigma_r = \sigma_r(1s)(1.20 + 0.28\alpha). \quad (3)$$

Relation (3) is well satisfied in the range  $\alpha = 0-3$ ,<sup>[3]</sup> corresponding to the temperature region  $T_e \gtrsim Z^2 \times 1.5$  eV. Taking (2) and (3) into account, we obtain:

$$\langle \sigma_r v_e \rangle = Z f(x), \quad x = 1/T_e, \quad (4)$$

$$f(x) = Ax^{3/2} \int_0^\infty \frac{e^{-xu^2}}{(u^2 + 1)^2} \frac{e^{-\frac{4}{u} \arctan u}}{1 - e^{-2\pi/u}} (1.20u + 0.28) du, \quad (5)$$

$$A = \frac{1024}{3} \pi \sqrt{\pi} \frac{e^2}{mc^2} \frac{e^2}{\hbar} \frac{\hbar}{mc} = 4.524 \times 10^{-12} \text{ cm}^3 / \text{sec}. \quad (6)$$

We integrated Eq. (5) numerically. Figure 1 shows the results of our calculation of  $\langle \sigma_r v_e \rangle$  for hydrogen (curve 1) as a function of  $T_e$ , as well as the values of  $\langle \sigma_r v_e \rangle$  calculated by formula (1) (curve 2). It is seen that both methods of calculating  $\langle \sigma_r v_e \rangle$  agree well in the region  $T_e < 32I$ . As to our calculation, it is valid in the case of recombination of any ion with ionized 1s shell. The validity of the calculation at high temperatures is restricted only by the need for introducing relativistic corrections.

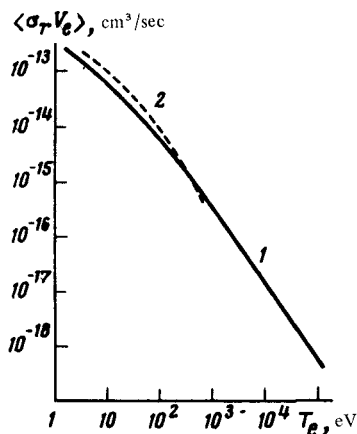


FIG. 1. Dependence of the hydrogen recombination rate on the plasma electron temperature. Curve 1—our calculation, 2—calculation by formula (1).<sup>[2]</sup>

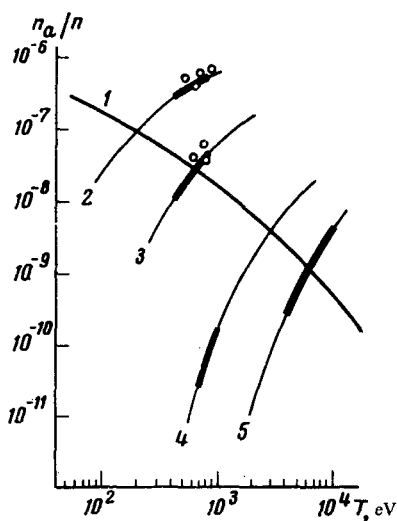


FIG. 2. Ratio of the atom concentration to the plasma concentration in the central region of the plasma pinch of a tokamak as a function of the plasma temperature: 1—atoms of recombination origin, 2—charge-exchange atoms in Tokamak-4 ( $n=6 \times 10^{13} \text{ cm}^{-3}$ ,  $a=17 \text{ cm}$ ), 3—charge-exchange atoms in Tokamak-10 ( $n=10^{14} \text{ cm}^{-3}$ ,  $a=35 \text{ cm}$ ), 4—charge-exchange atoms in "Alcator" ( $n=10^{15} \text{ cm}^{-3}$ ,  $a=8 \text{ cm}$ ), 5—charge-exchange in reactor tokamak ( $n=10^{14} \text{ cm}^{-3}$ ,  $a=150 \text{ cm}$ ). The thick segments on curves 2—5 show the ranges of the working temperatures of the corresponding tokamaks. The circles on curves 2 and 3 show the experimental atom concentrations determined from the atom flux emerging from the plasma.<sup>[1,8]</sup>

The calculated values of  $\langle \sigma_r v_e \rangle$  can be used to estimate the concentration of the hydrogen atoms produced in the hot plasma of thermonuclear installations by radiative recombination:

$$\frac{n_a^r}{n} = \frac{\langle \sigma_r v_e \rangle}{\langle \sigma_i v_e \rangle + \langle \sigma_i v_i \rangle}. \quad (7)$$

Here  $n$  is the plasma density,  $\langle \sigma_i v_e \rangle$  is the rate of ionization of the hydrogen atoms by the electrons,  $\langle \sigma_i v_i \rangle$  is the rate of ionization of the hydrogen atoms by the protons, and  $v_i$  is the relative velocity of the atoms and protons. Using the known data on the rates of ionization by electrons<sup>[4]</sup> and protons<sup>[5]</sup> we can plot the ratio  $n_a^r/n$  against the plasma temperature  $T$ . Such a plot is shown in Fig. 2 (curve 1). It is of interest to compare the atom concentrations  $n_a^r$  with the concentration  $n_a$  of the atoms that have penetrated into the plasma as a result of cascade charge exchange. Solution of the kinetic equation for the distribution functions of the tokamak-plasma atoms traveling from the surface into the interior of the plasma pinch<sup>[6]</sup> yields the following expression for the atom concentration  $n_a(0)$  on the plasma-pinch axis:

$$n_a(0) = \frac{1.23 \times 10^{10}}{\sqrt{T}} \exp \left( -\frac{6 \times 10^{-14} n a}{\sqrt{T}} \right). \quad (8)$$

Here  $a$  is the radius of the cross section of the plasma pinch of the tokamak, in cm. Expression (8) was derived by using the fact that the atom density on the plasma pinch boundary is  $n_a(1) = 10^{10} \text{ cm}^{-3}$  (a typical tokamak value obtained in numerous experiments). It is assumed that the atoms are incident on the plasma surface with energy  $\sim 2 \text{ eV}$ . The temperature dependence of the ratios  $n_a/n$  for modern tokamaks, and also for the large reactor tokamak presently under design (see, e.g., <sup>[7]</sup>), are shown in Fig. 2 (curves 2—5). Figure 2 shows that tokamaks of Tokamak-4 size operate in regimes wherein the atom concentration is determined mainly by the gas exchange between the plasma and the walls. In Tokamak-10, and also in the reactor tokamak, the recombination can play a noticeable role in the production of atoms in the plasma. In the American tokamak "Alcator," in which the plasma density is relatively high ( $n \approx 10^{15} \text{ cm}^{-3}$ ), <sup>[9]</sup> the atom concentration in the plasma is determined almost entirely by recombination.

Thus, the plots of Fig. 2 show that when the material and energy balance of the plasma of modern tokamak is considered, and account is taken of the role of the atoms in both balances, it is necessary to take into account in a number of cases the radiative recombination of the hydrogen

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