Possible appearance of the baryon asymmetry of the universe in an electroweak theory

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A new mechanism is proposed for the generation of the baryon asymmetry of the universe in an electroweak theory. This mechanism involves an anomalous nonconservation of baryon number at high temperatures. A cosmological limitation on the mass of a Higgs boson is derived: 10 GeV $\lesssim m_H \lesssim$ 60 GeV. The sign of the baryon asymmetry is determined by the sign of the CP breaking in the decays of K^0 mesons.

The appearance of the baryon asymmetry of the universe is usually linked with grand unification theories. However, there is no experimental evidence for grand unification, and we need to seek other mechanisms for the generation of the baryon asymmetry. In the present letter we propose such a mechanism, based on the existence of fast electroweak processes with an anomalous nonconservation of baryon number at high temperatures[1] and on the extremely probable degeneracy of the ground state of gauge theories with respect to the Chern-Simons number at $T \neq 0$. From the condition that the predicted baryon asymmetry of the universe must agree with the observed asymmetry we can extract a cosmological limitation on the mass of a Higgs boson: 10 GeV $\lesssim m_H \lesssim$ 60 GeV.

The numbers of baryons and of antibaryons are not the same because of the existence of fast processes involving a nonconservation of baryon number, CP breaking (described in this case by a Kobayashi-Maskawa matrix), and slight deviations from a thermodynamic equilibrium due to the expansion of the universe.[2] Analysis of a kinetic equation for the baryon chemical potential yields

$$\mu_B \sim bT^2\delta_{ms}/M_0. \tag{1}$$

Here T is the temperature of the universe; $M_0 = M_{Pl}/1.66N^{1/2}$ (N is the number of effectively massless degrees of freedom); δ_{ms} is the microscopic asymmetry in processes which do not conserve B, given by

$$\delta_{ms} = (B_{in} - B_{out})\ \frac{\sigma(in \to out) - \sigma(\overline{in} \to \overline{out})}{\sigma(in \to out) + \sigma(\overline{in} \to \overline{out})},$$

and b is the coefficient of the β-function for the SU(3) group: $\alpha_s(\mu) = 4\pi/b \ln \mu^2/\Lambda^2$. Deviations from a thermodynamic equilibrium are a consequence of a violation of conformal invariance due to a temperature dependence of the gauge coupling constant.

In the presence of a nonvanishing density of fermions, the effective action for the gauge fields acquires an increment[3]

$$\Delta E \sim -\mu_B N_{CS},\ \ N_{CS} = -\frac{1}{16\pi^2}\,\mathrm{Tr}(F_{ij}A_k - \frac{2}{3}A_iA_jA_k)\epsilon^{ijk}; \tag{2}$$

where N_{CS} is the density of the Chern-Simons number, and A_i are the gauge fields of the SU(2) group. (We have not written out the contribution proportional to the Abelian field B; it can be shown that this contribution has only a negligible effect on the generation of a baryon asymmetry of the universe.)

Because of the power-law infrared divergences in gauge theories at[4] $T \neq 0$, their ground state is nontrivial. Arguments based on perturbation theory indicate that this state contains a nonvanishing weak magnetic field $H \sim g_W^3 T^2$ with inhomogeneities of size scale[4] $(g_W^2 T)^{-1}$ [g_W is the gauge constant of the SU(2) group]. In the absence of a fermion density, a ground state of this sort would be degenerate with respect to a change in the Chern-Simons number density[1)]: $N_{CS} = N_{CS}^0 \cos\varphi$, $0 < \varphi \leqslant 2\pi$. In order of magnitude, we have $N_{CS}^0 \sim \alpha_W/\pi g_W^4 T^3$. The presence of a chemical potential μ_B lifts this degeneracy, and the state with the maximum (or minimum) value of N_{CS} becomes the preferred state in the case $\delta_{ms} > 0$ ($\delta_{ms} < 0$). Consequently, after a sufficiently long time has elapsed, the system is characterized by the maximum value of $|N_{CS}|$. After the electroweak phase transition, the gauge fields acquire a mass, and the nontrivial ground state becomes unstable. Its decay results in the appearance of an excess of baryons over antibaryons; the magnitude of this excess is equal to the Chern-Simons number density by virtue of the anomaly in the baryon current.[6] The sign of the asymmetry is determined by CP breaking in the decays of K^0 mesons.

The time dependence of N_{CS} in an expanding universe is described by

$$\frac{|N_{CS}(T)|}{N_{CS}^0(T)} = 1 - \frac{|N_{CS}(T_0)|}{N_{CS}^0(T_0)}\,\frac{2}{z}\,J_1(z),\quad z = 4(bg_W^2\alpha_W\delta_{ms}M_0/T)^{1/2}, \tag{3}$$

where $J_1(z)$ is the Bessel function of the first kind. In the case $z \gtrsim 1$, the quantity $|N_{CS}|$ executes damped oscillations around N^0_{CS}; in the case $z \ll 1$, on the other hand, the average value of the Chern-Simons number density is given in order of magnitude by $|N_{CS}| \sim z^2 N^0_{CS}/8$.

The final expression for the baryon asymmetry which arises in the decay of the ground state is

$$\Delta \equiv \frac{n_B}{n_\gamma} \cong \frac{N_{CS}(T_c)}{\frac{2\pi^2}{45} N T_c^3} S, \tag{4}$$

where $N_{CS}(T_c)$ is the Chern-Simons number density at the time of the electroweak phase transition, and S is the factor by which the asymmetry is suppressed (the decrease in the asymmetry due to B-nonconserving processes and entropy production during the phase transition is taken into account here). In the absence of heavy fermions, S would depend on only the mass of Higgs boson. It can be shown that we have $S(m_H) \cong 1$ at 10 GeV $\lesssim m_H \lesssim$ 56–60 GeV. Near $m_H \lesssim$ 10 GeV (this is the same as the mass of the Higgs boson in the theory with the Coleman-Weinberg potential[7]), we have[8] $S \cong 10^{-5}$–10^{-6}. This result is attributed to entropy generation during the phase transition. On the other hand, if $m_H \gtrsim$ 56–60 GeV, processes which do not conserve baryon number would still be at thermodynamic equilibrium, so that the baryon asymmetry which arose would be erased. The slight uncertainty in this estimate stems from our ignorance of the coefficient (β) of the exponential function in the expression for the rate of B-nonconserving processes below the critical point[1]: $v_B(t) = \beta T \exp[-3M_W(T)/\alpha_W]$. Values 56–60 GeV are found by varying β over the interval 10^{-3}–1. The cosmological constraint on m_H is therefore 10 GeV $\lesssim m_H \lesssim$ 56–60 GeV.

Major difficulties arise in attempts to determine the sign and magnitude of δ_{ms}, since the CP breaking in processes that do not conserve baryon number arises only in 12th-order perturbation theory in the Yukawa coupling constants:

$$\delta_{ms} \sim \left(\frac{g_W^2}{M_W^2}\right)^6 s_1^2 s_2 s_3 \sin\delta\, m_t^4 m_b^4 m_c^2 m_s^2 A \sim 10^{-21} A, \tag{5}$$

where $s_i = \sin\theta_i$, θ_1 is the Cabibbo angle, δ is the CP breaking phase, and the factor A incorporates the number of diagrams and their value for unit coupling constants (here we have used[9] $s_1^2 s_2 s_3 \lesssim 3\times 10^{-4}$). There are some significant uncertainties in the estimate of A, since $0(10^4)$ seven-loop diagrams contribute to A. A rough estimate yields $A \sim 10^{-1}$–10^5; the smallest number is found when fluctuations of the scalar field, $\langle \varphi^+ \varphi \rangle \sim T^2/3$, and the expectation value of the virtuality of the quark lines, $\langle p^{-1} \rangle \sim 1/T$, are taken into account, while the largest number is found under the assumption that the vacuum expectation value of the scalar field is $\varphi \sim g_W T$ at high temperatures.[10] We thus find $|\Delta| \sim (10^{-12}$–$10^{-4})$ not far from the actual value, $\Delta \sim 10^{-8}$–10^{-10}.

If the condition $\Delta\delta_{ms} \lesssim 10^{-21}$ actually holds in the standard model with three generations of fermions, cosmology will require the introduction of new particles in the theory. For example, the existence of a fourth heavy generation of fermions with a mass $\sim M_W$ would give us $\delta_{ms} \sim 10^{-12}$–$10^{-6}$; in theories with two Higgs doublets (including the case of supersymmetric doublets) one would naturally obtain values δ_{ms}, which lead to $z \gtrsim 1$ and thus to the generation of a maximum baryon asymmetry of the universe $\sim 0(10^{-4})S$. In this case, the baryon asymmetry which arises might be greater than that which is observed. A decrease in the baryon asymmetry would be possible only near $m_H \approxeq 10$ GeV and $m_H \approxeq 56$–60 GeV (in the absence of large Yukawa coupling constants and scalar coupling constants). Only these two values are permissible from the standpoint of cosmology if the condition $\delta_{ms} \gtrsim 10^{-21}$ holds.

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[1)]We do not rule out the possibility that a corresponding degeneracy occurs in the case in which an electroweak plasma has confinement properties at distances $\gtrsim (g_W^2 T)^{-1}$ (Ref. 5).

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