Spontaneous breaking of gauge symmetry in a homogeneous isotropic universe of the open type

A. A. Grib and V. M. Mostepanenko

Leningrad Institute of Precision Mechanics and Optics
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It is shown that spontaneous breaking of gauge symmetry takes place in the theory of a self-acting scalar field, considered in the open Friedmann model. It is concluded that the mass acquired by the photon by the Higgs mechanism becomes appreciable within nuclear times.

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Cosmological and astrophysical consequences of the assumption of spontaneous breaking of gauge symmetry of a scalar field have been actively investigated of late (see, e.g., [1–3]). Spontaneous symmetry breaking is introduced in the corresponding models in analogy with the Goldstone model[4] with the aid of the assumption that the bare mass is imaginary ($m^2 < 0$). The shortcomings of this assumption are well known, and have led to attempts to introduce into the theory spontaneous symmetry breaking without the use of this assumption.[5]

As shown in[6], spontaneous breaking of gauge symmetry arises when a complex scalar field with $m^2 > 0$ interacts with an external electric field of special types. We shall show here that spontaneous breaking of gauge symmetry arises when a zero-mass scalar field interacts with the gravitational field corresponding to the open Friedmann model. Allowance for the electromagnetic interaction causes the photon to acquire a nonzero mass in accordance with the Higgs mechanism.[7] From the estimates presented below it follows that at $t \sim 10^{-26}$ sec (the time t is reckoned from the singular state) this mass is equal in order of magnitude to the mass of the intermediate vector boson of the unified theories of weak and electromagnetic interactions, i.e., the electromagnetic interaction becomes short-range. The latter can lead to a change in the notions concerning the mechanism of certain physical processes during the earlier stages of the evolution of the universe.

We consider a self-acting scalar field $\Phi(x)$ with zero mass in a homogeneous isotropic universe of the open type. The space-time metric is

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = a^2(\eta)[d\eta^2 - d\chi^2 - \text{sh}^2\chi(d\theta^2 + \sin^2\theta d\phi^2)]. \tag{1}$$

As shown in[8–10], a correct generalization of the scalar wave equation with zero mass to include Riemannian geometry is the conformally-invariant equation

$$\Box\Phi(x) + \frac{R}{6}\Phi(x) + \frac{\lambda}{3}\Phi^*(x)\Phi^2(x) = 0, \tag{2}$$

where $\Box - \nabla^\mu\Delta_\mu, \nabla_\mu$ is the covariant derivative, R is the scalar curvature of space-time, $dt = a d\eta$, $c = \hbar = 1$.

The density of the Lagrangian corresponding to Eq. (2) is invariant to the one-parameter group of gauge transformations $\Phi \to \Phi \exp(i\alpha)$, $\Phi^* \to \Phi^* \exp(-i\alpha)$. Let $|0\rangle$ be a Heisenberg vacuum state defined, in analogy with[10], at the instant $t=0$. Taking into account the homogeneity of the metric (1) and the C-invariance of the state $|0\rangle$ we have

$$<0 \mid \Phi(\eta, x) \mid 0> = <0 \mid \Phi(\eta, 0) \mid 0> = g(\eta) = g^*(\eta). \tag{3}$$

Averaging (1) over the state $|0\rangle$ and neglecting in analogy with[4,11] the vacuum fluctuations, we arrive at the Duffing equation

$$\ddot{f}(\eta) - f(\eta) + f^3(\eta) = 0, \tag{4}$$

where the function $f(\eta)$ is connected with the function $g(\eta)$ introduced in (3) by the relation

$$g(\eta) = \sqrt{\frac{3}{\lambda}}\, \frac{f(\eta)}{a(\eta)}\ . \tag{5}$$

It is known that the zero-order solution of (4) is unstable. Consequently there are realized for (4) stable solutions $f(\eta) = \pm 1$, corresponding to the presence of spontaneous breaking of gauge symmetry.

We determine now the energy density and the pressure of the field Φ in an asymmetrical vacuum state $|0\rangle$. The metric energy-momentum tensor of the field $\Phi(x)$ is[9]

$$T_{\mu\nu}(x) = T^{can}_{\mu\nu}(x) - \frac{1}{3}\left[R_{\mu\nu} + \nabla_\mu \nabla_\nu - g_{\mu\nu} \Box\right] \Phi^*(x) \Phi(x). \tag{6}$$

Using (3)—(6) we obtain the negative values

$$\epsilon(\eta) = <0|T^\circ_\circ(x)|0> = -\frac{3}{2\lambda a^4},\quad P(\eta) = -<0|\,T^i_i(x)|0> = -\frac{1}{2\lambda a^4}, \tag{7}$$

satisfying the condition of conservativity and the equation of state $P = \epsilon/3$ (there is no sum over i). Negative ϵ at all t means that for a zero-mass field the state with spontaneously broken symmetry is energywise favored during all stages of evolution of the universe (it can be shown that in the case of a massive field $\epsilon < 0$ only up to a certain instant $t_0 \lesssim t_{pl}\, 10^{-43}$ sec.

If we rewrite the action S corresponding to (2) in a space that is conformal to the initial space with a static metric tensor $\tilde{g}_{\mu\nu} = a^{-2} g_{\mu\nu}$, then we obtain the action of the Goldstone model.[4] The only difference is that the 3-space specified by g_{ik} has a Lobachevskiĭ geometry, and its constant negative curvature plays the role of the negative square of the mass.

Let now the field Φ interact also with the massless vector field A_α. Regarding the result obtained above as the zeroth approximation in a unitary gauge we have

$$<0|\,\Phi'|0> = i<0|\,\Phi|0> = i\sqrt{\frac{3}{\lambda}}\,\frac{f}{a},\ \text{where}\ \Phi' = \frac{\Phi'_1 + i\Phi'_2}{\sqrt{2}},\ \Phi'_1 = 0 \tag{8}$$

(the field variables in the unitary gauge are primed).

Writing down the action of the considered system in terms of the field

$$\chi(x) = \Phi_2'(x) - \sqrt{\frac{6}{\lambda}}\,\frac{f(\eta)}{a(\eta)},$$

we obtain after transformations and using (8)

$$S = {}^1/_2 \int d^4x \sqrt{-g}\,\{g^{\alpha\beta}\frac{\partial\chi}{\partial x^\alpha}\frac{\partial\chi}{\partial x^\beta} - (m_\chi^2 + \frac{R}{6})\chi^2 + e^2 g^{\alpha\beta} A'_\alpha A'_\beta\,\chi^2$$

$$- {}^1/_2 F'_{\alpha\beta}F'^{\alpha\beta} + m_V^2 g^{\alpha\beta}A'_\alpha A'_\beta - \sqrt{\frac{2\lambda}{3}}\,\frac{f}{a}\chi^3 - \frac{\lambda}{12}\chi^4 + 2e^2\sqrt{\frac{6}{\lambda}}\,g^{\alpha\beta}A'_\alpha A'_\beta\chi, \qquad (9)$$

where $F'_{\alpha\beta} = \partial^\alpha A'_\beta - \partial^\beta A'_\alpha$, e is the charge of the electron, $m_\chi^2 = 3/a^2$, $m_V^2 = 6e^2/\lambda a^2$.

At the present stage of the evolution of the universe we have $a \sim 10^{28}$ cm and, assuming $\lambda \sim 1$, we have for the photon mass the negligibly small quantity $m_V \sim 10^{-66}$ g. At $t \lesssim 10^{-26}$ sec we obtain, however, $m_V \gtrsim 10^{-22}$ g, i.e., the electromagnetic interaction becomes short-range. This can lead to significant astrophysical consequences. For example, if $\Phi(x)$ is a pseudoscalar field interacting with a fermion field, then deviation of (3) from zero means P- and CP-parity violation which becomes appreciable at the indicated times.

We note in conclusion that, in contrast to[11], in which space-time of constant curvature is considered, the effect of spontaneous breaking of the symmetry does not depend here on the sign of $R/6$.

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