Acceleration of particles captured by a strong potential wave with a curved wave front in a magnetic field

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Particles can be accelerated without restriction by a potential wave with a wave front in the form of a rotating surface. The particles execute multiple revolutions along the azimuthal direction, making it possible to propose a more compact acceleration system than those that have previously been considered. The scale energies and energy of fast particles are found.

1. Acceleration of particles by a wave in a magnetic field parallel to the wave front[1–5] is a fundamental phenomenon with broad applications. This mechanism is the principle upon which the accelerator-serfotron, which makes use of strong laser and microwave radiation fields, is based.[3] This mechanism is also cited in the discussion of generation of fast particles at the wave fronts of collisionless shock waves under conditions corresponding to those occurring in space.[6] Quintessentially, this mechanism can be described as follows. In a coordinate system moving with the phase velocity of the wave, $v_{\rm ph}$, there arises an electric field $E \approx \beta_{\rm ph} B/\sqrt{1-\beta_{\rm ph}^2}$ which is directed along the wave front and which accelerates the particle. The ratio of displacement of the particle along the front, Δy, to the distance it traverses along with the wave, Δx, in the limit $\mathscr{E} \gg mc^2$ is $\Delta y/\Delta x = \sqrt{1-\beta_{\rm ph}^2}/\beta_{\rm ph}$. For $\beta_{\rm ph} \equiv v_{\rm ph}/c \ll 1$ the ratio $\Delta y/\Delta x$ is large, imposing considerable requirements on the size of the acceleration region and on the uniformity of the wave amplitude and the magnetic field.

2. The requirement imposed on the size of the region in which the acceleration is in the direction of the front is lifted if the front is of the same form as the rotation surface,

$$r_{\rm ph}(z,t) = v_{\rm ph} t - \frac{1}{2}\alpha(t) z^2 + \ \dots \ . \tag{1}$$

The Lagrangian for the motion along the surface (1) is

$$L^*(z,\varphi,\dot z,\dot\varphi,t) = -\, mc^2 \left(1 - \frac{\dot r_{\rm ph}^2 + 2\dot r_{\rm ph} r'_{\rm ph} \dot z + r_{\rm ph}^2 \dot\varphi^2 + (1 + r'^2_{\rm ph})\dot z^2}{c^2}\right)^{1/2}$$

$$+ \frac{e\dot\varphi}{c} \int_0^{r_{\rm ph}} B_z(r) r dr . \tag{2}$$

The magnetic field has only the z component, the dot denotes the derivative with respect to t, and the prime denotes the derivative with respect to the z coordinate. The generalized momentum is conserved because of the φ symmetry,

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$$p_\varphi = r_{\rm ph}^2 (\gamma\dot\varphi + \omega_z(r_{\rm ph})) = \text{const}, \quad \omega_z(r_{\rm ph}) = \frac{e}{mcr_{\rm ph}^2}\int_0^{r_{\rm ph}} B_z(r) r\,dr\,. \tag{3}$$

Here $\gamma = 1/(1-v^2/c^2)^{1/2}$ is the relativistic factor.

For the initial conditions we have $z(0)=0$, $z(0)=0$; the trajectory lies in the plane $z=0$, and as $t\to\infty$, it is given by

$$\dot\varphi = -\frac{\omega_z(r_\Phi)}{\gamma}, \qquad \gamma = \sqrt{\frac{1+\dfrac{\omega_z^2(r_{\rm ph})r_{\rm ph}^2}{c^2}}{1-\beta_{\rm ph}^2}}\,. \tag{4}$$

If $B_z = \text{const}$, in the ultrarelativistic limit $\gamma = e|B_z|r_{\rm ph}/2\sqrt{1-\beta_{\rm ph}^2}$, the trajectory has the shape of a logarithmic spiral, $\varphi = (\sqrt{1-\beta_{\rm ph}^2}/\beta_{\rm ph})\ln r$, and in the nonrelativistic case it has the shape of an Archimedes spiral, $\varphi = eB_z r/mcv_{\rm ph}$.

3. If the deviation from the $z=0$ plane is small, we can write, by virtue of (1), the following expression for the motion along the z coordinate:

$$\ddot z + \frac{\dot\gamma}{\gamma}\dot z - \frac{\alpha\omega_z^2(r_{\rm ph})v_{\rm ph}t}{\gamma^2}\left(1+\frac{d\ln\omega_z(r_{\rm ph})}{d\ln r_{\rm ph}}\right)z = 0, \tag{5}$$

where $\gamma(t)$ is given by expression (4). The stability of the motion is determined by the sign of the curvature of the front in the $\varphi = \text{const}$ plane (by the α sign). If the radius of the curvature, $R = 1/\alpha$, increases $R \approx v_{\rm ph}t$, we would have $z(t)\propto t^{(c-v_{\rm ph})/v_{\rm ph}}$ in the ultrarelativistic limit as $t\to\infty$. For a constant curvature ($\alpha>0$) we would have $z(t)\propto\exp(\sqrt{\alpha ct/\beta_{\rm ph}})$. For $\alpha<0$ the trajectory $z=0$ would be stable. It thus follows that acceleration at the wave front with a positive curvature ($\alpha>0$) is not as effective as acceleration at the front with a negative curvature ($\alpha<0$).

Some particles can nevertheless acquire a considerable amount of energy over a limited time, even when $\alpha>0$. The energy will increase as the time the trajectory remains near the $z=0$ plane is increased. This time in turn is determined by the degree to which the initial values of z and $\dot z$ differ from zero. If the wave front has finite dimensions along the z axis, of order z^*, the particle, having a finite energy, will escape from the acceleration region. This energy of the particle can be determined if we assume that in the ultrarelativistic limit it is proportional to the time the particle spends in the acceleration region. Hence, we can infer that $\mathscr{E}\propto(z^*/z_0)^{v_{\rm ph}/(c-v_{\rm ph})}$ for $\alpha\propto 1/v_{\rm ph}t$ and $\mathscr{E} = \mathscr{E}_1\ln(z^*/z_0)$ for $\alpha = \text{const}$, where $\mathscr{E}_1 = mc^2\beta_{\rm ph}\omega_z/2\alpha c\sqrt{1-\beta_{\rm ph}^2} = e|B_z|R/2\sqrt{1-\beta_{\rm ph}^2}$.

The differential energy spectrum of particles is proportional to $|dz_0/d\mathscr{E}|$ by virtue of the conservation of particle flux in the phase space.[7] We finally find that for $\alpha = 1/v_{\rm ph}t$ the spectrum is a power-law spectrum,

$$dN/d\mathscr{E}\propto\mathscr{E}^{-c/v_{\rm ph}}, \tag{6}$$

and for $\alpha = \text{const}$ $(\alpha > 0)$ the spectrum is an exponential spectrum

$$dN/d\mathcal{E} \propto \exp(-\sqrt{\mathcal{E}/\mathcal{E}_1}). \tag{7}$$

4. To find the condition under which a particle is held at the wave front, we must substitute relations (4) into the r component of the equations of motion in the coordinate system $r = r_{ph}(t)$ near $z = 0$:

$$\dot{\gamma} v_{ph} = \frac{e}{m} E(r_{ph}) + \frac{eB_z(r_{ph})}{mc} r_{ph}\dot{\varphi} + \gamma r_{ph}\dot{\varphi}^2. \tag{8}$$

Here $E(r_{ph})$ is the electric field. The last term describes the centrifugal-acceleration component. From (8) it follows that the condition for unrestricted acceleration, similar to that found in Refs. 3–5 for a plane wave, for the cylindrical case is

$$E(r_{ph}) \geqslant \frac{r_{ph}^2 B_z(r_\Phi) - \int_0^{r_{ph}^2} B_z(r) r\,dr}{r_{ph}^2\sqrt{1-\beta_{ph}^2}}. \tag{9}$$

If $B_z = \text{const}$, we will have $E > B_z/2(1-\beta_{ph}^2)^{1/2}$.

5. We assumed above that the magnetic lines of force are straight lines. In a simple model describing the curvature of the lines of force we have $\mathbf{B} = B_z\mathbf{e}_z - hz\mathbf{e}_r$, where B_z and h are constants, and B_z/h represents the radius of curvature of the lines of force. This effect leads to a modification of Eq. (5), in which α should be replaced by $\tilde{\alpha} = \alpha + 4h(1-\beta_{ph}^2)/B_z$. If $\tilde{\alpha} > 0$, the trajectory $z = 0$ is unstable and if $\tilde{\alpha} < 0$, the trajectory is stable.

6. If the ultrarelativistic particle moves along the wave front of a cylindrical wave, the rate of energy loss due to radiation will be $\dot{\mathcal{E}}_- = -(e^4B_z^2/6m^2c^3)(\mathcal{E}/mc^2)^2$ (see Ref. 8), and the rate at which the energy is acquired, by virtue of (4), will be $\dot{\mathcal{E}}_+ = (e|B_z|v_{ph}/2\sqrt{1-\beta_{ph}^2})$. Equating the acceleration rate to the loss rate, we find that the radiation loss limits the energy of fast particles to

$$\mathcal{E}_{max} = mc^2\left(\frac{6m^2c^3v_{ph}}{e^3B_z}\right)^{1/2} \approx mc^2\left(\frac{v_{ph}}{\omega_z r_e}\right)^{1/2} \gg mc^2, \tag{10}$$

where $r_e = e^2/mc^2$ is the classical electron radius.

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