Self-excitation of ballooning tearing modes; nature of the anomaly in the toroidal magnetic confinement of plasmas

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A self-excitation of tearing modes associated with the plasma pressure has been found. This self-excitation leads to an unconditional instability of these modes in toroidal systems with a strong longitudinal field. The results thus reveal a general mechanism, which the present authors regard as the primary mechanism, for the configurational disruption which determines the loss of energy from tokamak and stellarator plasmas.

All tokamak experiments reveal a mechanism of anomalous loss of energy from the plasma, which is manifested both during ohmic heating and during auxiliary heating, regardless of the heating method. Although the universal appearance of this effect indicates that the anomaly is of an MHD nature, we do not yet have an adequate theory of its mechanism. In the present letter it is shown that in a hot toroidal plasma, in which the condition $\mathbf{B}\widehat{\nabla}p$ holds well, the MHD perturbations of the tearing-mode type are always unstable because of the pressure gradient. The associated splitting of rational magnetic surfaces is, in our opinion, responsible for the anomalous transport in toroidal systems.

To illustrate the instability of ballooning tearing modes,[1] we consider the widely used simple model of a tokamak configuration with circular magnetic surfaces[1,2]: $g_{11} = 1, g_{12} = 0, g_{22} = a^2, g_{33} = (R - a\cos\theta)^2$. To describe both the linear and nonlinear characteristics of the MHD perturbations, we proceed in accordance with the theory of a nearby equilibrium. The tearing-mode perturbations split a resonance surface, converting into a chain of magnetic islands. The primary effect here is a flattening of the pressure profile along the width of an island ($2w$) because of the equilibrium condition $\mathbf{B}\widehat{\nabla}p = 0$. We take this effect into account in our model, flattening out the initial pressure profile along a band of width $2w$ near the resonant surface. Noting that the equations of a near equilibrium outside the islands are the same as Euler's equations in the theory of stability, we can find w as an eigenvalue from the condition for neutral stability of the modified pressure profile, as we did in Ref. 2.

We consider modes with large wave numbers $m,n \gg 1$. Introducing the dimensionless variable $x = m(a - a_m)|a_m$, $nq(a_m) = m$, and writing a trial plasma-displacement function in the form

$$\xi(x, \theta, \zeta) = e^{im\theta - in\zeta} \sum_l F_l(x) e^{il\theta}, \quad l = -1, 0, 1, \tag{1}$$

we find the following expression for the potential energy:

$$W = \sum_l \int_{-\infty}^{\infty} dx \left\{ [(l - Sx)F_l]'^2 + (l - Sx)^2 F_l^2 + \left(\frac{1}{2}\alpha^2 + \alpha U\right) F_l^2 + \frac{\alpha}{2} F_l' \left(F_{l+1} - F_{l-1}\right) - \alpha F_l F_{l+1} \right\} \tag{2}$$

where $S = q' a_m |q$ is the shear, $U = (1 - 1/q^2)a/R$ is the magnetic well, and the profile $\alpha(x) = -8\pi p' R q^2/B_s^2$ is shown in Fig. 1 for the quasilinear treatment. Assuming S, $\alpha \ll 1$ for simplicity, we have the Euler equations

$$S^2 \frac{d}{dx} x^2 \frac{dF_0}{dx} - S^2 x^2 F_0 = \left(\frac{\alpha^2}{2} + \alpha U\right) F_0 - \frac{\alpha'}{2} \bar{D} - \alpha(D + \bar{D}'),$$

$$D'' - D = -\frac{\alpha}{2} F_0, \quad \bar{D}'' - \bar{D} = \frac{\alpha'}{4} F_0 + \frac{\alpha}{2} F_0',$$

where

$$D = (F_1 + F_{-1})/2, \quad \bar{D} = (F_1 - F_{-1})/2.$$

The solutions for D and $\bar{D}$ are

$$D = \frac{1}{4} \int_{-\infty}^{\infty} \alpha F_0 e^{-|x-t|} dt,$$

$$\bar{D} = \frac{1}{4} \int_{-\infty}^{\infty} \alpha F_0 H(x - t) e^{-|x-t|} dt + \frac{1}{8} \int_{-\infty}^{\infty} \alpha' F_0 e^{-|x-t|} dt, \tag{3}$$

where $H(x) = 1$ at $x > 0$ and $H(x) = -1$ at $x < 0$. Substituting them into Eq. (3) for F_0, and after the standard procedure, we find the following expression for the energy:

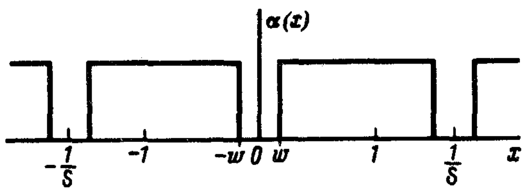

FIG. 1. The profile $\alpha(x)$ for a particular model for the effect of the pressure flattening caused by magnetic islands.

$$W = S^2 \int_{-\infty}^{\infty} [(xF_0)'^2 + x^2F_0^2]\,dx + \int_{-\infty}^{\infty} \alpha U F_0^2 dx - \frac{1}{16}\int_{-\infty}^{\infty} \alpha' F_0 dx \int_{-\infty}^{\infty} \alpha' F_0 e^{-|x-t|} dt$$

$$- \frac{1}{4}\int_{-\infty}^{\infty} \alpha' F_0 dx \int_{-\infty}^{\infty} \alpha F_0 H(x-t) e^{-|x-t|} dt. \qquad (4)$$

Now adopting the very simple trial function $F_0(x) = e^{-|x|}/x$, corresponding to a tearing-mode perturbation, and taking the profile $\alpha(x)$ into account, we find

$$W = S^2 \int_{-\infty}^{\infty} [(xF_0)'^2 + x^2F_0^2]\,dx + 2\int_{w}^{\infty} \alpha U F_0^2 dx + \alpha^2 F(w)\mathrm{ch}w \left[\int_{w}^{\infty} F_0 e^{-x} dx - \right.$$

$$\left. - \frac{1}{4} e^{-w} F_0(w)\right] = 2S^2 + 2\alpha U/w + \alpha^2\left(-\ln 2w - 0{,}6 - \frac{1}{4w}\right)/w, \qquad (5)$$

where the asymptotic behavior in the limit $w \to 0$ has been written out explicitly.

It is not difficult to see that if w is small the potential energy will always be negative, and even a magnetic well will be incapable of stabilizing the situation. It follows from the condition for neutral stability, $W = 0$, that the width of a saturated island can be estimated from

$$w = \frac{\alpha}{4U + \sqrt{16U^2 + 8S^2}}, \qquad (6)$$

and that it increases with increasing plasma pressure (Fig. 2). The transformation to a dimensional value of w is made in accordance with the rule $w \to a_m w/m$, from which we see that when all the wave numbers m,n are taken into account the magnetic islands should be overlapped, and the magnetic lines of force should be interwoven. This effect could not fail to increase the transport in hot plasma. It is important to note that, although the formulation of the problem of the stability of a modified pressure arose from a quasilinear analysis of the splitting of the magnetic field structure associated with the finite amplitude of the perturbation, the instability which has been found does not have a threshold in the amplitude. At a high temperature, where the condition $\mathbf{B}\nabla p = 0$ holds well, the growth rate of the instability should have an intermediate value, between the reciprocals of the Alfvén time and the skin time.

FIG. 2. Width of a saturated island versus the pressure gradient. 1—$S = 0.1$, $U = 0$; 2—$S = 0.1$, $U = 0.2$; 3—$S = 1.0$, $U = 0$; 4—$S = 1.0$, $U = 0.2$.

Our analysis shows that for ballooning tearing modes the smooth unperturbed pressure profile $\alpha(x) = \text{const}$, which is generally used in the theoretical work [α' is ignored in (3)], does not correspond to the physics of the instability, even in the approximation of small amplitudes, $w^2 \to 0$. Correspondingly, we cannot use the standard stability theory which ignores the effect of the perturbation on the pressure profiles being analyzed. This specific feature of ballooning tearing modes ("g-modes" in the old terminology) means that their instability may be regarded as a new MHD effect, which we call "self-excitation."

The self-excitation of ballooning tearing modes is basically related to the toroidal coupling of the harmonics of a perturbation, but otherwise it is independent of the nature of the toroidal system, even under the assumption $m,n \gg 1$. Its invariance with respect to the model of the plasma and the absence of a threshold in the pressure gives us every reason to assume that the disruption of the magnetic configuration which is caused by ballooning tearing modes is the mechanism which primarily determines the universally observed anomaly in the energy loss from toroidal systems with a strong longitudinal field (tokamaks and stellarators).

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[1]H. Strauss, Phys. Fluids **24**, 2004 (1981).
[2]L. E. Zakharov, S. B. Semenov, and K. Riedel, Pis'ma Zh. Eksp. Teor. Fiz. **44**, 26 (1986) [JETP Lett. **44**, 30 (1986)].

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