Effect of diffraction on absorption of lower hybrid waves in a tokamak

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(Submitted 9 September 1986)
Pis'ma Zh. Eksp. Teor. Fiz. **44**, No. 9, 426–428 (10 November 1986)

Diffraction effects cause a pronounced slowing of lower hybrid waves as they propagate through a tokamak plasma. The result is a sharp increase in the efficiency of the absorption of the waves by electrons.

In the experiments presently being carried out on current drive by lower hybrid waves in tokamaks, currents of several hundred kiloamperes have been attained, demonstrating the high efficiency of the interaction of the waves with electrons. On the other hand, no explanation of the mechanism for this interaction has been generally accepted. At the plasma boundary, traveling waves are usually excited with a phase velocity at least an order of magnitude higher than the electron thermal velocity. The wave absorption observed experimentally indicates that as the wave propagates through the plasma, there is a spectral pumping of wave energy toward lower phase velocities.

The ray method (the geometric-optics approximation) is used widely to solve problems involving the propagation of lower hybrid waves in plasmas.[1,2] It has been established in this method that a substantial slowing of the waves is possible only as a result of a repeated transit of the wave from the periphery of the plasma to its center and back. In the present letter we show that diffraction effects, which are not described by the basic approximation of the ray method, lead to a substantial broadening of the phase-velocity spectrum of the waves. Even in a single transit along the radius, the spectrum acquires some fairly slow waves which are capable of interacting efficiently with electrons. The physical reason for the strong effect of diffraction on the propagation of lower hybrid waves is that these waves are propagating in the form of narrow beams in a tokamak plasma,[3,4] and the amplitude of these beams falls off rapidly in the direction perpendicular to the beam axis. The transverse dimension of wave beam may be comparable to the wavelength; this situation will result in a diffractive spreading of the beam and will make the geometric-optics approach of only limited applicability in describing lower hybrid waves.

For simplicity, we restrict the discussion to the case in which the longitudinal slowing of the lower hybrid waves is substantial, $N_{\parallel}^2 \gg 1$, and we can use a potential approximation. The wave potential φ satisfies the wave equation

$$\operatorname{div} \hat{\epsilon} \operatorname{grad} \varphi = 0, \tag{1}$$

where $\hat{\epsilon}$ is the dielectric tensor of the plasma. We assume that the magnetic surfaces in a poloidal cross section of the tokamak are concentric nested circles. We introduce the quasicylindrical coordinates (r,θ,ζ), where r is the distance from the magnetic axis, θ is the poloidal angle, and ζ is the toroidal angle. By virtue of the toroidal symmetry of the system, we can seek the wave potential in the form $\varphi(r,\theta,\zeta,t) = u(r,\theta) \times \exp(in\zeta - i\omega t)$. We are interested in those solutions of Eq. (1) which describe narrow wave beams. The procedure for deriving such a solution is described in Ref. 5 and can be summarized as follows. We assume that a curve $\theta = \bar{\theta}(r)$ is a characteristic of wave equation (1). A solution localized near this characteristic is sought as a power series in a small parameter proportional to $\omega^{-1/2}$. In our problem, it is convenient to use the quantity $n^{-1/2}$ as this parameter, where n is the toroidal wave number (the condition $n > 100$ usually holds in a tokamak). It can be shown that a solution of Eq. (1) within terms of order $n^{-3/2}$ is

$$u(r,\theta) = N(r) H_l\left(\frac{\theta - \bar{\theta}(r)}{\sigma(r)}\right) \exp\left\{ -\frac{1}{2}\left(\frac{\theta - \bar{\theta}(r)}{\sigma(r)}\right)^2 + in\psi_l(r,\theta)\right\}, \tag{2}$$

where $H_l(r)$ is the Hermite polynomial, and $N(r)$ is a normalization factor. The wave phase $\psi_l(r,\theta)$ can be written as a power series in $\theta - \bar{\theta}(r)$, but since we will have no need of phase information below we will not reproduce the explicit expression for $\psi_l(r,\theta)$ here. The function $\sigma(r)$ is the solution of the differential equation

$$\frac{r^2k_r^3}{n^2\beta^2h_0^2}\frac{d}{dr}\left(\frac{r^2k_r^3}{n^2\beta^2h_0^2}\frac{d\sigma}{dr}\right)+\sigma\left[\frac{r^2k_r^3}{n^2\beta^2h_0^2}\frac{d}{dr}\left(\frac{rk_\theta h_1}{h_0}\right)\right.$$

$$\left.-\frac{r^2k_r^2h_2}{h_0^2}\left(\frac{rk_\theta}{nqR_0}+h_0\right)\right]=\frac{1}{\sigma^3}. \qquad (3)$$

Here $k_r(r)$ and $K_\theta(r)$ are components of the semiclassical wave vector $\mathbf{k}(r)$, which, like $\bar{\theta}(r)$, are determined by the equations of the characteristics. In experiments on current drive by lower hybrid waves, the inequality $\omega_{Be}^2 \gg \omega_{pe}^2(r) > \omega^2 > \omega_{pi}^2(r)$ usually holds. In this case we can write

$$\beta^2(r)=\frac{\omega_{pe}^2(r)-\omega^2}{\omega^2-\omega_{pi}^2(r)},\quad h(r,\theta)=\frac{1}{R_0-r\cos\theta},\quad h_k(r)=\left.\frac{\partial^k h(r,\theta)}{\partial\theta^k}\right|_{\theta=\bar{\theta}(r)},$$

where $q(r)$ is the safety factor, and R_0 is the distance from the major axis of the torus to the magnetic axis.

In the case $l=0$, the function in (2) describes the propagation of a Gaussian beam, and an arbitrary wave beam can be represented as a superposition of solutions of the form in (2). The function $\sigma(r)$ characterizes the beam localization region near the characteristic, $\theta=\bar{\theta}(r)$. The mean square value (averaged over the variable θ) of the half-width of mode l can be expressed in terms of $\sigma(r)$ by $\sqrt{l+0.5}\sigma(r)$. Equation (3) describes the change in the width of the wave beam during propagation into the plasma; it contains diffraction effects, which are not considered in the ray method. For example, it can be seen immediately from (3) that if the beam width at the plasma boundary is sufficiently small, the right side of the equation becomes the dominant part, and the function $\sigma(r)$ increases rapidly.

Lower hybrid waves are launched in tokamaks by a system of phased waveguides. Since the waveguides have a nonzero poloidal dimension, the wave spectrum will have a nonzero width in terms of the variable $N_\parallel$ even at a fixed value of n. Figure 2 shows the result of a solution of Eq. (3) for the parameter values of the Princeton Large Tokamak. This figure shows the change in the spectral width in $N_\parallel$ in the course of the propagation for two cases, in which the retardations at the boundary are $N_\parallel=2.1$ and $N_\parallel=4$. The dashed line shows the motion of the maximum of the wave packet along the radius [the curve $\theta=\bar{\theta}(r)$]. In order to interpret this figure, we should bear in mind that about 10% of the energy of the wavepacket is in modes $l \gtrsim 10$, for which the broadening of the spectrum is greater than that indicated in Fig. 1 by a factor $\sqrt{l+0.5}$. For the real spectra in the Princeton Large Tokamak, a broadening at this level is completely sufficient for the appearance of waves with $N_\parallel \geqslant 6$ and for the possibility of a resonant interaction of the waves with electrons.

In summary, diffraction increases the efficiency of the interaction of lower hybrid waves with a plasma by an order of magnitude or more, leading to a strong wave absorption even in the first transit along the radius. The method of ray paths as it is presently being used, does not describe this effect.

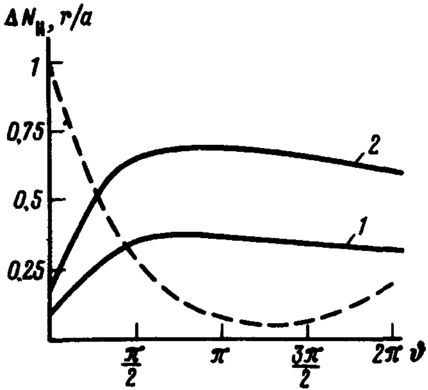

FIG. 1. Change in the spectral width ($\Delta N_{\parallel}$) of a packet of lower hybrid waves as they propagate through a plasma. The poloidal half-width of the wave beam at the plasma boundary is assumed to be $\sigma(a) = \pi/12$, $\omega_{pe}^2(r)/\omega^2 = 400 \quad [1-(r/a)^2]$; $q(r) = 1.5 + 2(r/a)^2$, $a = 40$ cm; $R_0 = 130$ cm. 1—$n = 70$ (this value corresponds to a retardation $N_{\parallel} = 2.1$ at the maximum of the packet); 2—$n = 130$, $N_{\parallel} = 4$.

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Translated by Dave Parsons