Gravitational interaction of massless high-spin ($s > 2$) fields

M. A. Vasil'ev and E. S. Fradkin
P. N. Lebedev Physics Institute, Academy of Sciences of the USSR, Moscow

(Submitted 22 October 1986)
Pis'ma Zh. Eksp. Teor. Fiz. **44**, No. 11, 484–488 (10 December 1986)

In contradiction of the general opinion, a noncontradictory gravitational interaction of massless fields of high spins $s > 2$ exists at least in the first nontrivial order. A fundamentally new aspect of the gravitational interaction of high spins is that it is nonanalytic in the cosmological constant.

An important problem in field theory is that of introducing a noncontradictory gravitational interaction of massless fields of high spins $s > 2$. Attempts to solve this problem were undertaken[1,2] immediately after the development of supergravity,[3–5] which solved the corresponding problem for spin 3/2. These attempts raised the hope that (first) it would become possible to overcome the well-known $N \leqslant 8$ barrier in an expanded supergravity, which was necessary in order to find the relativistic spectrum of low-spin particles in a theory of the pure supergravity type, and (second) it would be possible to achieve further cancellations of divergences in quantum gravity through the use of gauge symmetries of high spins.

We known quite well that all four-dimensional massless fields with spins $s \geqslant 1$ are gauge fields, and in the absence of an interaction they allow a systematic description in either a plane space[6–12] or an anti–de Sitter space.[13–15] It was shown in Refs. 1 and 2, however, that the introduction of a gravitational interaction of massless fields with spins $s > 2$ by making the free action covariant in a plane space is not systematic, since it leads to a breaking of gauge symmetries of high spins. We will show in this letter that a noncontradictory gravitational interaction of massless high spins does in fact exist in the first nontrivial order, i.e., in the same order in which the analysis was carried out in Refs. 1 and 2. A key factor here is that the gravitational interaction of massless high spins is not analytic in the cosmological constant, and it does not allow an expansion on a plane background, as was used in Refs. 1 and 2.

We will lean heavily on Refs. 11, 15, and 16, where a new form of the description of free massless fields on high spins in a plane space[11] and in an anti–de Sitter space[15] was proposed. In addition, an $N=1$ superalgebra of high spins was proposed there.[16] (Results analogous to those of Ref. 11 were obtained indepenently for spins 1/2 in Ref. 12.) Following Refs. 11 and 15, we associate a massless field of spin $s>1$ with a system of fields[1)] $\omega_{\nu,\alpha(n),\dot\beta(m)}$ with $n+m=2(s-1)$ [for fields $\omega_{\nu,\alpha(n),\dot\beta(m)}$, we will frequently use the notation $\omega(n,m)$]. The $N=1$ superalgebra of high spins[16] generates all spins $s>1$ (each spin is encountered once). The corresponding curvature and the corresponding infinitesimal gauge transformations with parameters ϵ are

$$R_{\nu\mu,\alpha(n),\dot\beta(m)} = \partial_\nu \omega_{\mu,\alpha(n),\dot\beta(m)} - \partial_\mu \omega_{\nu,\alpha(n),\dot\beta(m)}$$

$$+\sum_{p,q,s,k,l,t=0}^{\infty} i^{s+t-1}\frac{n!\,m!}{p!\,q!\,s!\,k!\,l!\,t!}\,\delta(n-p-q)\delta(m-k-l)$$

$$\times\lambda^{1+\frac{1}{2}(|n-m|-|p+s-k-t|-|q+s-l-t|)}\,\delta(|(p+k)(q+l)+(p+k)(s+t)+(q+l)(s+t)+1|_2)$$

$$\times\,\omega_{\nu,\alpha(p)\gamma(s),\dot\beta(k)\dot\delta(t)}\,\omega_{\mu,\alpha(q)}{}^{\gamma(s)}{}_{,\dot\beta(l)}{}^{\dot\delta(t)}, \tag{1}$$

$$\delta\omega_{\nu,\alpha(n),\dot\beta(m)} = \partial_\nu \epsilon_{\alpha(n),\dot\beta(m)} +$$

$$+\sum_{p,q,s,k,l,t=0}^{\infty} i^{s+t-1}\frac{n!\,m!}{p!\,q!\,s!\,k!\,l!\,t!}\,\delta(n-p-q)\,\delta(m-k-l)\,\times$$

$$\times\lambda^{1+\frac{1}{2}(|n-m|-|p+s-k-t|-|q+s-l-t|)}\,\delta(|(p+k)(q+l)+(p+k)(s+t)+(q+l)(s+t)+1|_2)$$

$$\times\,\omega_{\nu,\alpha(p)\gamma(s),\dot\beta(k)\dot\delta(t)}\,\epsilon_{\alpha(q)}{}^{\gamma(s)}{}_{,\dot\beta(l)}{}^{\dot\delta(t)}. \tag{2}$$

We use the notation

$$\delta(n)=\begin{cases}1 & \text{for } n=0\\ 0 & \text{for } n\neq 0,\end{cases}\quad \theta(n)=\begin{cases}1 & \text{for } n\geqslant 0\\ 0 & \text{for } n<0,\end{cases}\quad |n|_2=\begin{cases}1 & \text{for } n=2k+1\\ 0 & \text{for } n=2k\end{cases},$$

$$\epsilon(n)=\theta(n)-\theta(-n),\qquad |n|=n\theta(n)-n\theta(-n). \tag{3}$$

The gauge fields satisfy the conditions for a Hermitian nature, $(\omega_{\nu,\alpha(n),\dot\beta(m)})^\dagger = \omega_{\nu,\beta(m),\dot\alpha(n)}$, which correspond to the superalgebra $shs_0(1)$ in the notation of Ref. 16. These gauge fields are assumed to be (anti-) commuting if the number of spinor indices, $n+m$, is (not) even. The parameter λ is the same as the reciprocal radius of the anti–de Sitter space (the cosmological constant is proportional to $-\lambda^2$). The curvatures in (1) are found from curvatures (1.1) of Ref. 16 by transforming to the new fields

$$\omega_{\nu,\alpha(n),\dot\beta(m)} \to \lambda^{1-\frac{1}{2}|n-m|}\,\omega_{\nu,\alpha(n),\dot\beta(m)} . \tag{4}$$

This method for introducing λ is dictated by the requirement that the free theory and the linearized constraints (more on this below) must allow us to take the plane limit, $\lambda \to 0$, at finite fields $\omega(n,m)$. An important point is that the curvatures in (1) contain λ raised to not only positive but also negative powers. This circumstance ultimately makes the *interaction* of the high spins nonanalytic in the cosmological constant.

The gravitational tetrad and gravitational connection are identified with the fields

$$\begin{aligned}
\omega_{\nu,\alpha(1),\dot\beta(1)} &= h_{\nu\alpha\dot\beta} + \omega'_{\nu,\alpha(1),\dot\beta(1)} ,\\
\omega_{\nu,\alpha(2),\dot\beta(0)} &= w_{\nu\alpha(2)} + \omega'_{\nu,\alpha(2),\dot\beta(0)} ,\\
\omega_{\nu,\alpha(0),\dot\beta(2)} &= \bar w_{\nu\dot\beta(2)} + \omega'_{\nu,\alpha(0),\dot\beta(2)} ,
\end{aligned} \tag{5}$$

where it is assumed that the background tetrad h and the connections w, $\bar w$ are small quantities of zeroth order. The deviations of ω' of the gravitational fields from the background fields and all fields $\omega(n,m)$ with $n+m\neq 2$ are assumed to be small quantities of the first order. The background fields are chosen in such a way that the corresponding zeroth-order curvatures [$sp(4)$ curvatures], given by

$$r_{\nu\mu,\alpha(2)} = \partial_\nu w_{\mu\alpha(2)} + w_{\nu\alpha\gamma} w_{\mu\alpha}{}^{\gamma} + \lambda^2 h_{\nu\alpha\dot\delta} h_{\mu\alpha}{}^{\dot\delta} - (\nu \leftrightarrow \mu), \tag{6}$$

$$r_{\nu\mu,\alpha\dot\beta} = \partial_\nu h_{\mu\alpha\dot\beta} + w_{\nu\alpha\gamma} h_\mu{}^{\gamma}{}_{\dot\beta} + \bar w_{\nu\dot\beta\dot\delta} h_{\mu\alpha}{}^{\dot\delta} - (\nu \leftrightarrow \mu) \tag{7}$$

[$\bar r_{\nu\mu,\dot\beta(2)} = (r_{\nu\mu,\beta(2)})^\dagger$], vanish. This result means that the curvatures in (1) have an expansion $R = R^l + O[(\omega^{(\prime)})^2]$, where the linearized curvatures R^l are small quantities of first order, and $O[(\omega^{(\prime)})^2]$ means the terms of second order.

To describe the dynamics of high spins, we use the following invariant action, which clearly is generally covariant[2]:

$$S = \frac{1}{2}\sum_{n+m>0} i^{n+m+1}\frac{1}{n!\,m!}\,\beta(n+m)\in(n-m)\,\lambda^{-|n-m|} \times \int d^4x\, \epsilon^{\nu\mu\rho\sigma} R_{\nu\mu,\alpha(n),\dot\beta(m)} R_{\rho\sigma,}{}^{\alpha(n)}{}_{,}{}^{\dot\beta(m)} , \tag{8}$$

This is a direct generalization of the action of free, massless high spins in a de Sitter space which was proposed in Ref. 15 [if we replace the curvatures R by R^l in (8), then S becomes the sum of free actions for all spins $s>1$; $\beta[2(s-1)]$ is a normalization coefficient for the free action of spin s]. An important property of action (8) is that in the quadratic approximation its variation over all the "extra" fields $\omega(n,m)$ with $|n-m|>2$ vanishes identically.[15] As a result, the free high spins are described exclusively by "dynamic" $\omega(n,m)$ with $|n-m|\leqslant 2$. When the interaction is turned on, however, the variation of action (8) in terms of the extra fields is nonzero,[16] so that it cannot be interpreted in a reasonable way if the extra fields are assumed to be independent dynamic variables. A way out of this difficulty is to express all the extra fields from the outset in terms of dynamic fields by means of certain constraints. Constraints making this possible at a linearized level were proposed in Ref. 15; they are[3)]

$$\epsilon^{\nu\mu\rho\sigma} R^{l}{}_{\nu\mu,\alpha(n),\dot\beta(m-1)\dot\delta}\, h_{\rho\alpha}{}^{\dot\delta} = 0 \qquad \text{for} \qquad n \geqslant m > 0, \tag{9}$$

$$\epsilon^{\nu\mu\rho\sigma} R^{l}{}_{\nu\mu,\alpha(n-1)\gamma,\dot\beta(m)}\, h_{\rho}{}^{\gamma}{}_{\dot\beta} = 0 \qquad \text{for} \qquad m \geqslant n > 0. \tag{10}$$

Remarkably, as will be shown below, even the linearized constraints in (9) and (10) are sufficient to prove the invariance to action (8) in the cubic approximation.

The extra fields appear in (8) only in nonlinear combinations of the type $R^{l}\omega\omega$, so that it is sufficient to know their linearized expressions in the cubic approximation. Since we are interested in only terms of the type $R^{l}\omega\epsilon$ in the variation of the action, it is sufficient to know the transformation laws for the extra fields in simply the zero order. Since constraints (9) and (10) are proportional to the curvatures, a variation of these constraints under gauge transformations (2) is a small quantity of at least first order. Correspondingly, the deformed transformation laws of the extra fields, which are dictated by the requirement that the constraints be invariant, differ from (2) only beginning at first order. Consequently, in the present approximation we can use transformations (2) and constraints (9) and (10), forgetting about the noninvariance of the latter.[4)]

After constraints (9) and (10) are solved, all the fields $\omega(n,m)$ with $|n-m|>1$ are expressed in terms of the physical fields $\omega(n,m)$ with $|n-m|\leqslant 1$ with an accuracy to within a purely gauge part corresponding to linearized transformations (2) (explicit expressions are given in Ref. 15). The degree of the highest-order derivatives of the physical fields contained in (n,m) is $\frac{1}{2}(|n-m|-|n-m|_2)$. The extra fields thus introduce higher derivatives in the interaction of the high spins, including an interaction of these spins with gravitational curvatures.

A variation of action (8) under transformations (2) is

$$\delta S = \sum_{p,q,s,k,l,t=0}^{\infty} i^{p+q+s+k+l+t} \frac{1}{p!\,q!\,s!\,k!\,l!\,t!}$$

$$\times \lambda^{1-\frac{1}{2}(|p+q-k-l|+|p+s-k-t|+|q+s-l-t|)}$$

$$\times \delta(|(p+k)(q+l)+(p+k)(s+t)+(q+l)(s+t)+1|_2)$$

$$\times (\beta(p+q+k+l)\epsilon(p+q-k-l)-\beta(p+s+k+t)\epsilon(p+s-k-t))$$

$$\times \int d^4x\, \epsilon^{\nu\mu\rho\sigma} R_{\nu\mu,\alpha(p+q),\dot\beta(k+l)} R_{\rho\sigma,}{}^{\alpha(p)}{}_{\gamma(s),}{}^{\dot\beta(k)}{}_{\dot\delta(t)}\, \epsilon^{\alpha(q)\gamma(s)}{}_{,}{}^{\dot\beta(l)\dot\delta(t)}. \tag{11}$$

We can show that if the coefficients $\beta(n)$ are chosen appropriately, the variation in (11) vanishes on constraints (9) and (10) and the free equations of motion of the physical fields. A key point of the proof is the following assertion, which was itself proved in Ref. 15: If constraints (9) and (10) are satisfied, then the linearized curvatures R^l can be put in the form

$$R^l{}_{\nu\mu,\alpha(n),\dot{\beta}(m)} = \frac{i}{4}[\delta(m)h_\nu{}^\gamma{}_{\dot{\delta}}h_\mu{}^{\gamma\dot{\delta}}C_{\alpha(n)\gamma(2)} - \delta(n)h_{\nu\gamma}{}^{\dot{\delta}}h_\mu{}^{\gamma\dot{\delta}}\overline{C}_{\dot{\beta}(m)\dot{\delta}(2)}] + O\left(\frac{\delta S^2}{\delta\omega^p}\right), \tag{12}$$

where

$$C_{\alpha(n+2)} = \frac{1}{4h}\epsilon^{\nu\mu\rho\sigma}h_{\nu\alpha\dot{\delta}}h_{\mu\alpha}{}^{\dot{\delta}}R^l{}_{\rho\sigma,\alpha(n),\dot{\beta}(0)},$$
$$\overline{C}_{\dot{\beta}(m+2)} = \frac{1}{4h}\epsilon^{\nu\mu\rho\sigma}h_{\nu\gamma\dot{\beta}}h_\mu{}^\gamma{}_{\dot{\beta}}R^l{}_{\rho\sigma,\alpha(0),\dot{\beta}(m)}, \tag{13}$$

and $h = \det|h_\nu^{\alpha\dot{\beta}}|$ and $O(\delta S^2/\delta\omega^p)$ represents terms which are proportional to a variation of the free action of high spins with respect to the physical fields $\omega(n,m)$ with $|n-m|\leqslant 1$.

It is clear from (12) that it is also necessary to find the coefficients $\beta(n)$ at which (11) has no term of the type $C\times C$, $\overline{C}\times\overline{C}$, or $C\times\overline{C}$. Using the simple identity $\epsilon^{\nu\mu\rho\sigma}h_{\nu\dot{\delta}}^{\ \alpha}h_\mu^{\ \alpha\dot{\delta}}h_{\rho\gamma}^{\ \beta}h_\sigma^{\ \gamma\beta} = 0$, we easily see that there are no terms $C\times\overline{C}$ for arbitrary $\beta(n)$. The $C\times C$ and $\overline{C}\times\overline{C}$ terms are absent if and only if (11) has no terms which contain either curvatures with only point indices or curvatures with nonpoint indices, i.e.,

$$\delta(|pq+ps+qs+1|_2)[\beta(p+q)\theta(p+q-1) - \beta(p+s)\theta(p+s-1)] = 0 \tag{14}$$

for arbitrary $p,q,s\geqslant 0$. The general solution of (14) is $\beta(n) = \text{const} = \beta$ at $n>0$, where the choice of constant is dictated by the normalization of the gravitational action. In this case, variation (11) is proportional to $\delta S^2/\delta\omega^p$, so that we find those modified transformations of the physical fields, $\delta'\omega^p = \delta\omega^p + \Delta\omega^p$, which leave action (8) invariant in our approximation [$\delta\omega^p$ is given in (2), while $\Delta\omega^p$ has an $R^l\times\epsilon$ structure].

In summary, action (8) with $\beta(n) = \beta$, supplemented by constraints (9) and (10), gives a noncontradictory description of the dynamics of all massless fields with spins $s\geqslant 3/2$ in the cubic approximation. The spin-2 field is a gravitational field. The theory contains only two independent constraints: the gravitational constant and the cosmological constant.

[1)]We are using the following rules for inversion with indices.[15] The indices α, β, γ,... = 1,2 and $\dot{\alpha}$, $\dot{\beta}$, $\dot{\gamma}$,... = 1,2 are spinors. The indices of the components of the differential forms are ν, μ, ρ,... = 0–3. Over the spinor indices denoted by a single letter, the situation is first rendered completely symmetric separately in terms of the upper and lower indices; then a convolution of the maximum possible number of lower and upper indices denoted by a common letter is carried out. The number of indices is given in parentheses (except in the case of a single index).

[2)]Action (8) is a generalization of the MacDowell-Mansouri action[17] for a (super) gravity with a cosmological term: That part of action (8) which depends on only the fields $\omega(n,m)$ with $n+m=2$ ($1\leqslant n+m\leqslant 2$) is the same as the action of the pure gravity [$N=1$ supergravity with $\beta(1)=\beta(2)$].

[3)]With $n=m$, constraints (9) and (10) are the same as the linearized (algebraic) equations of motion of the auxiliary fields[15] $\omega(n,m)$ with $|n-m|=2$, which are analogous to a gravitational Lorentzian connection. The extra fields are eliminated by relations (9) and (10) with $n\neq m$.

[4)]Incorporating the deformation of the transformations of the physical fields, which is discussed below, does not change this conclusion. Furthermore, using the arguments in the spirits of the "formalism of order 1.5" (Refs. 18 and 19), we easily see that the modification of the transformation law for the auxiliary fields $\omega(n,m)$ with $|n-m|=2$ is also inconsequential in this approximation.

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Translated by Dave Parsons