Supersymmetry grand unification theory with an automatic fine adjustment

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A modified version of the $SU(5)$ supersymmetry theory of grand unification is proposed. In this version of the theory, the problem of the "light doublets" is solved automatically by means of the hypothesis which states that the Higgs sector has a higher symmetry than the $SU(5)$ symmetry. If the popular mechanism for the breaking of supersymmetry due to supergravity is assumed to be valid, we obtain a rigorously determined low-energy Lagrangian for the Higgs fields, which depends solely on a single weak-supersymmetry-breaking parameter—the gravitino mass $m_{3/2}$.

It is known that in supersymmetry grand unification theories the radiative corrections do not disrupt the hierarchy incorporated in the tree approximation.[1–3] On the other hand, this hierarchy is achieved only due to the fine adjustment of the superpotential parameters. For a most common type of superpotential in the $SU(5)$ theory, for example,

$$W = \frac{1}{2} M \mathrm{Tr}\Sigma^2 + \frac{\lambda}{3} \mathrm{Tr}\Sigma^3 + f(H_1 \Phi H_2) + m(H_1 H_2), \tag{1}$$

where $\Phi \sim 24$, $H_1 \sim 5$, and $H_2 \sim 5^*$, we must impose the condition

$$\lambda m = 3fM \tag{2}$$

in order to account for the (nearly) massless nature of the doublets of the electroweak group, which appear in the 5-plets of H_1 and H_2, for the massiveness ($\sim M$) of the corresponding color triplets. Condition (2) appears to be quite artificial. In the present letter we propose a version of the theory in which the doublets automatically remain massless and the triplets have a large mass, on the order of the unification mass.

We assume that the Higgs sector of the theory has a gauge symmetry higher than the $SU(5)$ symmetry: a global $SU(6)$ symmetry. All Higgs fields can be inserted in a natural way into the associated $SU(6)$-group representation by adding the $SU(5)$ singlet field φ: $35 = 1 + 5 + 5^* + 24$. In the supersymmetry version of the theory, the superpotential would then depend on a single chiral field, $\Sigma \sim 35$, and would have the form

$$W = \frac{1}{2} M \mathrm{Tr}\Phi^2 + \frac{\lambda}{3} \mathrm{Tr}\Phi^3. \tag{3}$$

For an arbitrary $SU(n)$ group a superpotential such as that in (3) would lead to a potential which would have several degenerate minima corresponding to a total energy of zero (unbroken symmetry) upon breaking the $SU(n)$ group to $SU(m) \times SU(n-m) \times U(1)$ $(m=1,...,n-1)$. It is generally assumed that all such minima are physically equally justifiable and we can assume that any one of them can occur in a true vacuum. We assume that the $SU(6)$ symmetry which we are considering is broken to $SU(4) \times SU(2) \times U(1)$. In this case we have

$$\langle \Sigma \rangle = \frac{M}{\lambda} \begin{pmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & -2 & \\ & & & & & -2 \end{pmatrix} . \quad (4)$$

The residual $SU(4)$ symmetry contains the gauge $SU(3)$ group, while $SU(2) \times U(1)$ is an ordinary electroweak group. It is obvious that the gauge $SU(5)$ symmetry, by virtue of (4), is broken to the standard $SU(3) \times SU(2) \times U(1)$ symmetry. From the standpoint of the $SU(5)$ classification, the quantity $\langle \Sigma \rangle$ in (4) corresponds to the following expectation values of the 24-plet $\langle \Phi \rangle$ and the singlet $\langle \varphi \rangle$:

$$\langle \Phi \rangle = \frac{M}{\lambda} \begin{pmatrix} 6/5 & & & & \\ & 6/5 & & & \\ & & 6/5 & & \\ & & & -9/5 & \\ & & & & -9/5 \end{pmatrix} , \qquad \langle \varphi \rangle = -\frac{M}{\lambda} \sqrt{\frac{6}{5}} . \quad (5)$$

This can clearly be seen from the decomposition of the 35-plet in $SU(5)$ multiplets

$$\Sigma = \begin{pmatrix} \dfrac{-5\varphi}{\sqrt{30}} , & H_1 \\ H_2 , & \Phi_{ij} + \dfrac{\delta_{ij}\varphi}{\sqrt{30}} \end{pmatrix} . \quad (6)$$

The residual $SU(4)$ symmetry includes, in addition to the color gauge symmetry, the global symmetry between the $SU(5)$ singlet and the color. There is a total of $35-(15+3+1)=16$ generators broken spontaneously upon breaking of the $SU(6)$ group. Of the 16 Goldstone bosons 12 are consumed by the Higgs mechanism because of the weighting of the $SU(5)$ gauge X and Y bosons. The remaining four massless Goldstone bosons are $SU(2)$ doublets in the 5-plets of H_1 and H_2. Since $\Sigma^+ \neq \Sigma$ and $H_2^+ \neq H_1$ in the supersymmetry theory, two independent complex doublets in H_1 and H_2 have eight real components, rather than four. It is easy to see that the doublet part of the combination $(H_1 + H_2^+)/\sqrt{2}$ is the true Goldstone boson, whereas the doublet from $(H_1 - H_2^+)/\sqrt{2}$ remains massless because of supersymmetry. Aside from the two massless doublets, all remaining fields have a mass of $\sim M$.

The method of breaking the supersymmetry must be indicated before a realistic low-energy theory can be formulated. We assume that the supersymmetry is broken due to the supergravity,[4,5] which seems to be the most popular hypothesis currently. The increment to the supersymmetric scalar-field potential can then be constructed according to the relation

$$\delta V = m_{3/2}^2 \,\mathrm{Tr}\Sigma^+\Sigma + A m_{3/2} \frac{\lambda}{3} (\mathrm{Tr}\Sigma^3 + \mathrm{Tr}\Sigma^{+3}) + (A-1) m_{3/2} \frac{M}{2} (\mathrm{Tr}\Sigma^2 + \mathrm{Tr}\Sigma^{+2}), \tag{7}$$

where $m_{3/2}$ is the gravitino mass, and A is a parameter of order 1, whose magnitude is determined by the hidden sector of the theory.[6-8] The trace in (7) is the trace of the matrix indices. [It is possible that because of the renormalization, the coefficients of the quadratic and cubic terms in (7) are independent coefficients. This circumstance has virtually no effect on the discussion below.] It is easy to see that the vacuum expectation value in (4) retains its structure but is multiplied by an additional factor:

$$1 + \frac{m_{3/2}}{M} + \frac{m_{3}^2}{M^2}(A-3) + O\left(\frac{m_{3/2}^3}{M^3}\right) . \tag{8}$$

As a result of combination of the doublet Higgs fields, $(H_1 - H_2^+)/\sqrt{2}$ acquires a mass of $m_{3/2}$, whereas the state $(H_1 + H_2^+)/\sqrt{2}$, a Goldstone boson, remains a rigorously massless state. (We retain for the doublet fields the same notation for H_1 and H_2 that we have previously used for the corresponding 5-plets.) Interestingly, the mass of the doublet $(H_1 - H_2^+)/\sqrt{2}$ does not depend on the parameter A; for the case in which the coefficients of the terms $\sim\Sigma^3$ and $\sim\Sigma^2$ in (7) are independent (A and B, respectively), this mass is $m_{3/2}\,\{2[1+(A-B)^2]\}^{1/2}$.

The total low-energy potential of the doublet fields H_1 and H_2 is

$$V = 4m_{3/2}^2 \frac{H_1 - H_2^+}{\sqrt{2}} \frac{H_1^+ - H_2}{\sqrt{2}} + D = \text{terms}$$

$$= 2m_{3/2}^2 [H_1^+ H_1 + H_2'^+ H_2' + H_1 \epsilon H_2' - H_2'^+ \epsilon H_1^+] + \frac{g^2}{8}(H_1^+ H_1 - H_2'^+ H_2')^2$$
$$+ \frac{g^2}{2}(H_1^+ H_2')(H_2'^+ H_1) , \qquad H_2' = \epsilon H_2 . \tag{9}$$

In the last equation we introduced the doublet $H_2' = \epsilon H_2$, instead of the antidoublet H_2.

The Lagrangian of the type in (9) was studied extensively.[9-13] The mass terms in this Lagrangian are generally assumed to depend on several parameters. Specifically, the coefficient of $[H_1 \epsilon H_2' + \mathrm{H.c.}]$ is not the same as the coefficient of $[H_1^+ H_1 + H_2'^+ H_2']$. In our model the structure of the low-energy Lagrangian is rigorously determined. Equation (9), is not, however, a definitive equation. It is valid at high energies on the order of the unification mass M. The renormalization-group equation can be used to obtain an actual potential.[9-13] If the coupling constant of the doublet H_2' ($Y = +1$) with a t quark is large, there can be a change in the sign of the mass parameter for $H_2'^+ H_2'$ and there can be a spontaneous violation of the standard $SU(2)\times U(1)$ group. A detailed analysis of the renormalization-group equations is beyond the scope of this paper. We might note that upon engagement of the gauge interactions the doublet $(H_1 + H_2^+)/\sqrt{2}$ undoubtedly acquires a mass, since this state is no longer a Goldstone boson because the gauge interactions do not have an $SU(6)$ symmetry.

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