Quantum model of technicolor with composite gauge bosons

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A quantum model of technicolor is proposed. In this model the composite gauge bosons, a Higgs meson, and new vector particles arise from a bosonization of techniquark currents. Relations between the masses are derived. The Weinberg angle is also derived.

In the technicolor model of Ref. 1, the Higgs meson is a composite particle, while the gauge bosons W_μ and B_υ are assumed to be elementary. In the quantum model of technicolor proposed in the present letter, all the bosons of the standard Weinberg-Salam-Glashow electroweak theory are composite particles. In addition, this new model contains several new bosons, whose existence follows from the group structure of the model. The primary difficulty in introducing composite gauge fields is in achieving gauge invariance in the absence of nucleating gauge fields in the original Lagrangian. Pursuing the analogy with the method of an invariant Fock space,[2] we construct a kinematic gauge potential from a composite field. The direction of the Abelian (electromagnetic) rotation in techni-isospin space is identified with the direction of the technipion field.

The technifermion Lagrangian and the quantum functional of the model are

$$\mathcal{L}_\psi = i\bar{\psi}\gamma^\mu(\partial_\mu + G_\mu)\psi \equiv \bar{\psi}\not{D}\psi,$$
$$Z_\psi(\not{D}) = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp i\int \bar{\psi}\not{D}\psi, \qquad (1)$$

where $G_\mu = G_\mu^a T_a$ is the technicolor field, and T_a are anti-Hermitian generators of the technigroup. Each techniquark has two techniflavors, so that the techni-isospin group is $SU(2)$. This group has no elementary field. It is assumed that the technicolor regime is similar at low energies to quantum chromodynamics; specifically, it is characterized by chiral, $C_q = \langle\bar{\psi}\psi\rangle$, and positive technigluon, $C_g = (\alpha_s/\pi)\langle G_{\mu\nu}^2\rangle$ constants.[3]

Composite boson fields arise as parameters of the transformation matrix $\exp\omega$ of techniquark fields, $\psi \to \exp\omega\,\psi$, or $\not{D} \to \exp\omega^+ \not{D} \exp\omega$, which changes the quantity Z_ψ (Ref. 4). Although we are interested in vector and pseudovector and also pseudoscalar techni-isospin fields which are singlet in the technicolor, we introduce all ten types of fields on the basis of group considerations. These are the coefficients in the expansion of ω in the total system of 8×8 matrices constructed on the basis of Dirac and isospin matrices:

$$\omega = \bar{\sigma} + \bar{\Pi} + \bar{\rho} + \bar{A}_5 + \bar{F},$$
$$\bar{\sigma} = \sigma_0\tau_0 + \hat{\sigma},\ \bar{\Pi} = \gamma_5(\Pi_0\tau_0 + \hat{\Pi}),\ \bar{F} = \sigma_{\mu\nu}(F_0^{\mu\nu}\tau_0 + \hat{F}^{\mu\nu}),$$
$$\bar{\rho} = \not{\rho}_0\tau_0 + \hat{\not{\rho}},\ \bar{A}_5 = i\gamma_5(\not{A}_0\tau_0 + \hat{\not{A}});\ \hat{A} = A_a\tau_a,\ \not{A} = \gamma^\mu A_\mu. \qquad (2)$$

The effective action $W_{\rm eff}(\omega)$ is not invariant under gauge transformations $S(x)$: $W_{\rm eff}(\omega) = -i\ln Z_\psi(\not{D})Z_\psi^{-1}(e^{\omega^+}\not{D}e^{\omega}) \neq W_{\rm eff}(\omega^S)$, $\omega^S = S\omega S^+$, $SS^+ = 1$.

Among fields (2), only the technicolor field $\bar{\Pi}$ is massless. We determine the gauge space $SU(2)\times U(1)$ in terms of the field $\bar{\Pi}$, and we identify the direction of the Abelian (electromagnetic) rotation in the $SU(2)$ space with the direction $\hat{\Pi} = \hat{n}\Pi$. We can then introduce gauge fields and make the theory Abelian-invariant. We introduce the matrix $\alpha(x) = U\exp i(\Pi\tau_3 + \Pi_0)$, which is given by the field Π; here $\hat{\Pi} = U\tau_3U^+\Pi, UU^+ = 1$. Under the gauge transformation $\alpha \to S\alpha$, in particular, under the Abelian transformation $S_n = \exp(i/2)(\hat{n} + y)\varphi$, we have $\Pi \to \Pi + \varphi/2$, and $\Pi_0 \to \Pi_0 + y^\varphi/_2$. The ratio $\Pi_0/\Pi = y$ characterizes the techniquarks; y is a $U(1)$ charge, while $Q = \frac{1}{2}(\hat{n} + y)$ is the electric charge. The field $\bar{\alpha}\partial_\mu\bar{\alpha}^+$ determines the vector gauge transformation in the space of techniflavors. We assume that W_μ and B_μ are gauge fields of the $SU(2)$ and $U(1)$ groups, so that $L_\mu = W_\mu + 1/2yB_\mu$ transforms as $\bar{\alpha}\partial_\mu\bar{\alpha}^+$. The behavior of $R_\mu = \frac{1}{2}(\hat{n} + y)B_\mu + U\partial_\mu U^+$ is analogous.

The parameters in (2) are related to the gauge fields by

$$\hat{A}_{5\mu} = \frac{i}{2}(L_\mu - R_\mu),\ -iF_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu + [V_\mu, V_\nu],\ V_\mu = \frac{1}{2}(L_\mu + R_\mu).$$

The fields $\bar{\rho}$, $\hat{\sigma}$, and $\not{A}_{50}$ do not depend on the gauge fields; $\bar{\rho}$ and $\hat{\sigma}$ transform homogeneously; and A_{50} describes a neutral pseudovector isosinglet. The Higgs field H_β is $H_\beta = \overline{\alpha}_{\beta 2} \exp \sigma_0$, since the potential part of the effect of Lagrangian for H^+H (known from quantum chromodynamics[3]) has the form $(H^+H) = (C_g/24) \operatorname{tr}[\frac{1}{2}(H^+H)^2 \ln H^+H]$, which is characteristic of a spontaneous symmetry breaking. The weak hypercharge (y) of the techniquarks is equal to y(Higgs). The technipion field $\overline{\Pi}$ determines the gauge; in the unitary gauge we have $\overline{\Pi} = \Pi_0 = 0$. In general, we should transform to $\dot{\omega} = \overline{\alpha}^+ \omega \overline{\alpha}$, where all fields undergo only an Abelian transformation, e.g., $\dot{\overline{\Pi}} = \gamma_5 \times (\tau_3 + y)\overline{\Pi}$, $\dot{W}_\mu = \overline{\alpha}^+ W_\mu \overline{\alpha} + \overline{\alpha}^+ \partial_\mu \overline{\alpha}$, $\overline{A}_5 = \overline{\alpha}^+ \dot{\overline{A}}_5 \overline{\alpha}$. The effective Lagrangian $\mathscr{L}_{\rm eff}(\dot{\omega})$ is given in terms of the physical fields ω in the approximation quadradic in ω as follows (at this point, we drop the symbol from $\dot{\omega}$:

$$\mathscr{L}_{\rm eff}(\omega) = \frac{F_0^2}{4} \operatorname{tr} \{ (\partial_\mu \overline{\sigma})^2 + (\partial_\mu \overline{\Pi})^2 - \frac{2}{3} (\partial_\mu \overline{A}_5)^2 - \frac{2}{3} (\partial_\mu \overline{\rho})^2 + \frac{1}{3} (\partial_\mu \overline{F})^2 \}$$
$$- \frac{C_g}{6} \operatorname{tr} \{ \overline{\sigma}^2 - \frac{1}{4} \overline{\rho}^2 - \frac{3}{4} \overline{A}_5^2 + \frac{1}{6} \overline{F}^2 \} + \mathscr{L}_T, \tag{3}$$

where all the fields are dimensionless, F_0 is the technipion constant, and α_T is the tachyon term, given by

$$\mathscr{L}_t = \frac{n_f}{480\pi^2} \operatorname{tr} (3\sigma \not{\partial}^4 \sigma - 5\overline{\Pi} \not{\partial}^4 \Pi - 4\overline{\rho} \not{\partial}^4 \overline{\rho} + 2\overline{A}_5 \not{\partial}^4 \overline{A}_5). \tag{4}$$

Here n_f is the dimensionality of the technicolor representation of techniquarks. The cubic interaction without derivatives is

$$\mathscr{L}^{(3)} = \frac{1}{9} C_g \operatorname{tr} (\omega^3 + \frac{3}{4} \omega \gamma_\mu \omega^2 \gamma^\mu). \tag{5}$$

According to (3), the masses of all the particles are proportional to the technigluon condensate C_g: $m_W^2 = (3C_g/8F_0^2)$, $m_\rho^2 = (\frac{2}{3}) m_W^2 = (\frac{3}{8}) m_\sigma^2 = (\frac{1}{3}) m_A^2$. The Weinberg angle θ_W is determined by the weak hypercharge of the techniquarks (or of the Higgs meson): $\tan^2\theta_W = (1 + 2y^2)^{-1} = \frac{1}{3}$, $\theta_W = 30°$. The mass of the Higgs meson in our model is therefore ~ 110 GeV. Incorporating term (4) changes this value. The model contains two parameters, F_0 and C_g, which characterize the low-energy region of technicolor. These parameters can be fixed in terms of the mass of the W boson and the electric charge e^2 by means of the relations $3e^2F_0^2 = 4m_W^2$ and $C_g = 2e^2F_0^4$. With $e^2 \approx 0.1$ we have $F_0 \cong 300$ GeV and $C_g = (140 \text{ GeV})^4$. Accordingly, in addition to the bosons of the standard Weinberg-Salam-Glashow electroweak theory, this model predicts the existence of massive vector (nongauge) particles: an isotriplet and an isosinglet with a weak hypercharge $y_\rho = 0$ and identical masses m_ρ, a pseudovector-isosinglet with $y_A = 0$ and mass m_A, and an isotriplet of scalar mesons σ with $y_\sigma = 0$ and m_σ. The new particles, however, may not interact directly with leptons and quarks. Their interaction with other fields is described in the simplest case by triple vertices in (5).

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