Two-dimensional Langmuir collapse and two-dimensional Langmuir cavitons

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The particular features of the collapse of Langmuir waves in a two-dimensional geometry are discussed. The results of a simulation of the collapse kinetics are used to identify regions above the critical level corresponding to the two limiting cases of a pure collapse and of a quasistationary caviton and also to an intermediate case of the final stage of the evolution of a cavity.

1. The collapse of Langmuir waves (plasma waves), i.e., the formation in a plasma of regions of reduced density which undergo a catastrophic depending, and which contain trapped Langmuir oscillations, is a fundamental phenomenon in plasma physics (see, for example, the review by Zakharov[1]). This phenomenon was definitely observed in recent experiments.[2] As a result of the collapse, energy of the Langmuir oscillations is transferred to fast electrons, so that this collapse is an important mechanism (often the primary mechanism) for the damping of Langmuir waves in plasmas both in space and in the laboratory. Collapse is therefore an important physical effect.

An alternative to collapse might be the formation of cavitons: quasistationary, weakly damped plasma solitons. Numerical simulation is playing a major role in research on the conditions for the occurrence of collapse. These simulations have been carried out since[3] 1974, in two directions. Most of the work (including that of Ref. 3) has been aimed at solving an equation for the complex envelope of the high-frequency electric potential, averaged over the period of the Langmuir oscillations:

$$\Delta(i\psi_t + \frac{3}{2}\ \omega_p r_D^2 \Delta\psi) = \frac{\omega_p}{2n_0}\ \mathrm{div}\,(\delta n \nabla\psi). \tag{1}$$

A closed system of equations has been formed by supplementing the equation above with a linear wave equation for the low-frequency variation of the plasma density:

$$\left(\frac{\partial^2}{\partial t^2} - c_s^2\Delta\right)\delta n = \frac{1}{16\pi m_i}\ \Delta\,|\nabla\psi|^2. \tag{2}$$

Equations (1) and (2) have steady-state solutions—solitons or cavitons—for a space of arbitrary dimensionality d. Analysis of the linear approximation reveals stability for a one-dimensional soliton but an instability for a three-dimensional caviton. An important point is that a two-demensional caviton is in a state of neutral stability in the linear approximation, but it is unstable with respect to finite perturbations. In the dimensionality $d = 2$ or 3, Eqs. (1) and (2) describe a collapse, i.e., the formation of deepening density cavities, in which the field becomes infinite after a finite time $t = t_0$, for certain initial conditions. The collapse is a self-similar process asymptotically as $t \to t_0$; the cavity size L decreases in accordance with $L \sim (t_0 - t)^{2/3}$ for $d = 3$ and in accordance with $L \sim (t_0 - t)$ for $d = 2$. A collapse results, in particular, from the onset of an instability of a caviton. These conclusions are supported quite well by a numerical solution of (1) and (2).

Actually, of course, the field in a cavity remains finite. In order to stop the collapse, it is sufficient to take into account the "saturation of the nonlinearity": the slowing of the growth of the variation of the plasma density as the energy of the Langmuir oscillations at the center of the cavity increases. A suitable way to take this effect into account is to replace Eq. (2) by a kinetic equation for ions:

$$\frac{\partial f_i}{\partial t} + \mathbf{v}\frac{\partial f_i}{\partial \mathbf{r}} - \frac{e}{m_i}\nabla\varphi\frac{\partial f_i}{\partial \mathbf{v}} = 0, \quad \Delta\varphi = -4\pi e\left(\int f_i d\mathbf{v} - \bar{n}_e\right), \tag{3}$$

$$\bar{n}_e = n_0 \exp\left(\frac{e\varphi - \phi}{T_e}\right), \quad \phi = \frac{e^2}{4m_e\omega_p^2}|\nabla\psi|^2, \quad \delta n = \bar{n}_e - n_0.$$

According to system (1), (3) with $d = 2$, the cavitons acquire a certain margin of linear stability (a margin that shrinks rapidly with increasing size). Three-dimensional cavitons of sufficiently small scale ($kr_D \sim 1$) also become stable. According to system (1), (3), the collapse should therefore terminate in the formation of stable cavitons. As was shown in Ref. 4, this process should be accompanied by the emission of intense sound waves from the cavity. The deviation of Eq. (3) from Eq. (2) is seen particularly vividly in the two-dimensional case.

In order to finally resolve the question in favor of collapse or cavitons, it is necessary to determine how a rapidly collapsing cavity (or a forming soliton) transfers its energy to electrons. Equations (1)–(3) do not incorporate the interaction of the Langmuir waves with electrons in a fundamental way. A patchwork way to incorporate this interaction would be to include dissipative terms in Eq. (1). When dissipation is taken into account, the collapse of fairly intense wave packets would also be possible in the one-dimensional case.[5] A more systematic approach, however, is to carry out a direct simulation of the collapse by a particle method. Anisimov *et al.*[6] have demonstrated that this type of simulation is possible in principle.

2. We have carried a numerical simulation of a two-dimensional Langmuir collapse, making maximum use of the symmetry of the potential in the cavity with respect to the center of the cavity: $\psi(x,y) = \psi(-x,y) = -\psi(x,-y)$. In a first series of calculations, the simulation was carried out by the particle method on the square $0 < x < L_x/2$, $-L_y/2 < y < L_y/2$ ($L_x = 128r_D$, $L_y = 64r_D$) with reflection boundary conditions $\partial\psi/\partial n|_\Gamma = 0$. The square contained the right half of a cavity with dimensions L_x, L_y. In a second series we carried out a step-by-step simulation of Langmuir collapse (a "beginning-to-end calculation"). In the first step, average equations (1)

and (2) were solved in the region $0<x<L_x/2$, $0<y<L_y/2$, which corresponds to a fourth of the cavity, with the boundary condition $\psi|_{y=0}=0$ ($L_x = 1024r_D$, $L_y = 512r_D$). When the size of the cavity became small in comparison with the size of the region, we singled out its central part, $0<x<L_x/4$, $0<y<L_y/4$, which we then extended to cover the entire calculation region. The change in the energy of the cavity here was no more than 10%. After this process of singling out a central part was carried out twice, the resulting distributions of the field and the particle density and velocity were adopted as initial data for a second ("kinetic") step of the simulation, by the method of particles on a square $0<x<L_x/2$, $-L_y/2<y<L_y/2$ ($L_x/2=L_y=128r_D$). The ratio of the ion and electron masses was varied from 100 to 1836. The initial distribution of electrons was taken to be locally Maxwellian, while the ions were assumed to be initially cold. As the initial condition on system (1), (2) we selected a function ψ satisfying

$$\Delta\psi = -\lambda \sin \kappa_y y(1+\cos k_x x), \quad \kappa_y = \frac{\pi}{L_y}, \quad k_x = \frac{2\pi}{L_x}.$$

For the variation of the ion density at the initial time we selected

$$\left.\frac{\delta n}{n_0}\right|_{t=0} = -\frac{1}{16\pi n_0 T_e} |\nabla\psi|^2, \quad \left.\delta\dot{n}\right|_{t=0} = 0.$$

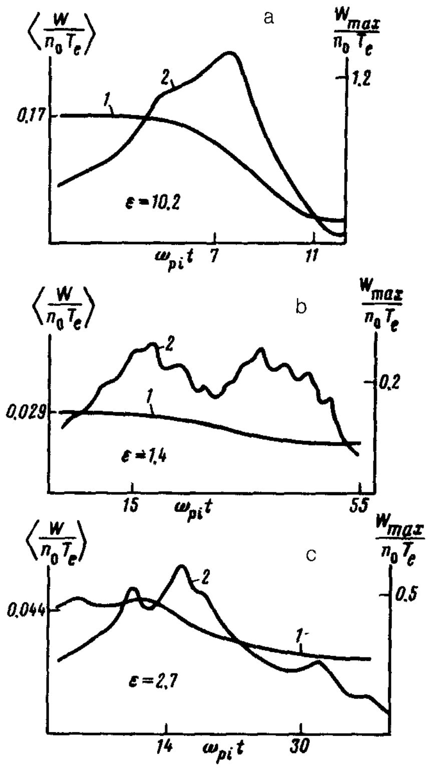

FIG. 1. a,b,c—Time evolution of the average energy (curves 1) and of the maximum energy (2) of the high-frequency field in the cavity for various values of ϵ.

The total number of particles of each species was $\sim 4\times 10^5$.

3. The calculations showed that the ratio of the particle masses causes a self-similar variation of the scale times of the process, which are conveniently expressed in dimensionless units: $\tau = \omega_{pi} t$. A qualitatively important quantity is the "supercriticality" parameter ϵ, which is the ratio of the initial energy of the high-frequency field in the cavity to the threshold energy for the modulational instability. In the case $\epsilon < 1$, the saturation of the instability, after the self-similar contraction, stabilizes the level of the high-frequency field in the cavity. The nature and duration of the subsequent absorption stage turn out to depend strongly on ϵ; $\tau(\epsilon)$ falls off rapidly with increasing ϵ. At $\epsilon \gg 1$, there is a rapid ($\epsilon = 10.2 \tau \sim 4$) and essentially complete (85%) absorption of energy (Fig. 1a), corresponding to a "pure" collapse. In the opposite case in which the threshold is slightly exceeded, the absorption stage is accompanied by oscillations of the level of the high-frequency field in the cavity. This stage lasts so long ($\epsilon = 1.4 \tau \sim 40$) that we can speak in terms of a quasistationary caviton (Fig. 1b). When the threshold for the modulations instability is exceeded to a moderate extent, we have an intermediate regime ($\epsilon = 2.7 \tau \sim 16$; $\epsilon = 2.1 \sim 20$), which could naturally be interpreted as a "drawn-out" collapse (Fig. 1c). At all values of ϵ, the cavity contracts to dimensions on the order of $11 \cdot 23 r_D^2$.

There are accordingly three characteristic regimes in the final stage of the evolution of a cavity, depending on the extent to which the threshold for the modulational instability is exceeded: a collapse, a "drawn-out" collapse, and a quasistationary caviton. We wish to emphasize that this result is determined by the particular features of the two-dimensional geometry of the problem, in which the saturation of the nonlinearity plays a much more important role than in the real three-dimensional case.

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