State-density oscillations of two-dimensional electrons in a transverse magnetic field

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A theory is derived for the nonlinear screening by a two-dimensional electron gas of the potential of charged impurities distributed at random in a volume. This theory is used to derive the position of the Fermi level and the width of the state-density peak at the Landau level as functions of the electron density in a magnetic field.

The state density of two-dimensional electrons in a magnetic field is usually regarded as consisting of a set of Landau levels separated from each other by $\hbar\omega_c$ and having a width $\Gamma \ll \hbar\omega_c$. If this width is related to the short-range potential, the state density falls off with increasing energy ϵ (reckoned from the Landau level) in accordance with $\exp(-\epsilon^2/\Gamma^2)$ and is very small halfway between levels. In Refs. 1 and 2,

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the state density was studied by studying the activation energy for the conductivity as a function of the degree of filling of Landau levels. It was found that the state density between levels is considerably higher than predicted by the estimate above. The width Γ has recently been measured on the basis of luminescence spectra.[3] It has been found that Γ oscillates, increasing sharply with decreasing $\delta n = |n - Mn_0|$, where n is the two-dimensional density of electrons, n_0 is the density corresponding to a complete filling of one Landau level, and M is an integer. This behavior implies that the state density is not of a one-electron nature. In this situation we should draw a distinction between the state densities found by different methods.[4] While Refs. 1 and 2 dealt with the quantity $D(E_F) = dn/dE_F$, where E_F is the Fermi energy, in Ref. 3 the state density was treated as a function of the energy at a fixed filling. When the electron-electron interaction is taken into account, these state densities should not be the same. Oscillations of Γ as a function of the filling were predicted in Ref. 5, where they were attributed to a periodic change in the screeing radius. No analytic expressions for Γ were given in Ref. 5. Furthermore, the screening was assumed to be linear there, although at small values of δn it is nonlinear, as we will show below.

Let us consider a two-dimensional electron gas in the $z = 0$ plane, surrounded by charged centers which are distributed at random in a thick layer between the planes $z = d$ and $z = -d$. We will derive the functions $D(E_F)$ and $\Gamma(\delta n)$ at small values of δn, finding results in qualitative agreement with experiment. We begin by considering the case of strong magnetic fields, with $\hbar\omega_c \gg e^2/\kappa a$, where e is the electron charge, κ is the dielectric constant, and a is the radius of the hydrogen-like state. In this case we may assume that none of the completely filled Landau levels participates in the screening and that the density of screening carriers at $\delta n \ll n_0$ is δn. When the filling is slight, the carriers are electrons, and when the filling is pronounced, they are holes. To describe the screening of a random potential with a length scale L, we partition the $z = 0$ plane into squares of size $L \times L$, and we use each of these squares as the base of an $L \times L \times L$ cube. The fluctuation in the number of charges in each such cube is on the order of $\sqrt{NL^3}$, where N is the concentration of centers. If $\sqrt{NL^3}$ is smaller than $\delta n L^2$, the number of electrons in an $L \times L$ square, i.e., if $L > L_c$, where

$$L_c \equiv \frac{N}{(\delta n)^2}, \tag{1}$$

a potential with a length scale L is screened by electrons. In the opposite case, $L < L_c$, screening does not occur. The length L_c is therefore a nonlinear-screening radius.[6] The amplitude of the random potential is determined by the scale L_c and is given in order of magnitude by

$$\Gamma = \alpha \frac{e^2}{\kappa L_c} \sqrt{NL_c^3} = \alpha \frac{e^2 N}{\kappa \delta n}, \tag{2}$$

where α is a numerical coefficient. According to (2), Γ increases rapidly in the limit $\delta n \to 0$. When Γ reaches a value on the order of $\hbar\omega_c$, however, two Landau levels begin to participate simultaneously in the screening, so that Γ stops increasing. This behavior agrees qualitatively with the results of Ref. 3. A competing limitation on Γ arises when L_c reaches d. Expressions (1) and (2) hold under the condition $NL_c^3 \gg 1, \delta n L_c^2 \gg 1$, i.e., under condition

$$(\delta n)^3 \ll N^2. \tag{3}$$

Condition (3) places a large-δn limit on the range of applicability of (2). If $n_0^3 \ll N^2$, expression (2) gives a correct estimate of Γ up to $\delta n \cong n_0/2$. Since the potential varies smoothly, the width of a luminescence line should be smaller than that calculated above.

To now calculate $D(E_F)$, we note that the Fermi energy is shifted by an amount on the order of Γ from the unperturbed Landau level by fluctuations of the potential. The Fermi energy reckoned from the nearest Landau level is therefore

$$E_F = \beta \frac{e^2 N}{\kappa \delta n}, \tag{4}$$

where β is a numerical coefficient. We then find

$$D(E_F) \equiv \frac{dn}{dE_F} = |\beta| \frac{e^2 N}{\kappa E_F^2} = \frac{(\delta n)^2 \kappa}{|\beta| e^2 N}. \tag{5}$$

We see that $D(E_F)$ increases rapidly with decreasing E_F. It should be kept in mind, however, that the range of applicability of (5) is limited by condition (3) and that the condition $\delta n \leqslant n_0/2$ holds. The state density $D(E_F)$ reaches a minimum at $E_F = \hbar\omega_c/2$, and we have

$$D(\hbar\omega_c/2) = 4|\beta| \frac{e^2 N}{\kappa(\hbar\omega_c)^2}. \tag{6}$$

In a gap between Landau levels, the state density $D(E_F)$ is not exponentially small, as it would be in the case of a short-range potentials. If charged centers were present only in the $z = 0$ plane, we would find $E_F = e^2 N_2^{1/2} \kappa^{-1} \ln(N_2/\delta n)$, where N_2 is the surface concentration of centers.[7] The state density $D(E_F)$ would then turn out to be exponentially small in the gap. For the state density $D(E_F)$ in the gap, the charged centers in the volume thus play a greater role than that played by surface centers. The reason is that centers in a layer of rapidly increasing thickness L_c become involved in forming the potential as δn decreases.

Up to this point, we have been discussing potential fluctuations with a length scale greater than $N^{-1/3}$. There is, however, another factor which tends to lower the Fermi level. This factor arises because of positive centers which by chance turn out to lie within a distance $z \ll N^{-1/3}$ from the $z = 0$ plane. If $E_F \leqslant e^2/\kappa a$, this component is on the order of (4), while at $E_F > e^2/\kappa a$ it is small in comparison with (4). This component is sharply suppressed if there is an undoped layer near $z = 0$.

The case which we have been discussing above is that with $\hbar\omega_c \gg e^2$, the case which prevails in a material with a sufficiently small effective mass. In silicon metal-insulator–semiconductor structures the inequality $\hbar\omega_c \ll e^2/\kappa a$ holds. We assume that in this case the results derived above remain qualitatively correct, although the analysis becomes more complicated. At $\hbar\omega_c \ll e^2/\kappa a$, there are fluctuations which bind electrons so strongly that the magnetic field has essentially no effect on their state. There are, on the other hand, fluctuations in which the binding energy of the electrons is small in comparison with $\hbar\omega_c$. An analysis of these fluctuations is essentially analogous to that above and leads to the same results.

The nonlinear-screening theory presented here can also be used to describe the state of two-dimensional electrons in a zero magnetic field, provided that the total density of electrons is so low that the inequality $n^3 \ll N^2$, analogous to (3), holds. In this case we need to replace δn by n in (1) and (2). We then find the activation energy for the electrical conductivity, ϵ_A, to be

$$\epsilon_A = \beta \frac{e^2 N}{\kappa n} . \qquad (7)$$

At values of N large enough to satisfy $Na^3 > 1$, a transition from an activated conductivity to a metallic conductivity occurs at a density $n = n_c =$ such that the condition $n_c^3 \ll N^2$ holds. To determine n_c, we should equate (7) to the Fermi energy of a degenerate two-dimensional gas, $\pi\hbar^2 n/m$. As a result, we find $n_c \propto \sqrt{N/a}$. At $Na^3 \ll 1$, the inequality $n^3 < N^2$ initially breaks down with increasing n, and a Wigner crystal or a Wigner liquid with a correlation energy greater than Γ arises throughout the space. The Wigner crystal or liquid will cause a linear screening along a distance $n^{-1/2}$. In this case the theory of the metal-insulator transition is more complicated.

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After completing this article, we learned that the nonlinear screening by a two-dimensional electron gas in the absence of a magnetic field had been studied by Gergel and Suris.[8]

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