

# Quantum oscillations of the density and Fermi energy of electrons at an inversion layer in a magnetic field

V. M. Pudalov and S. G. Semenchinskii

*All-Union Scientific-Research Institute of the Metrological Service*

(Submitted 23 October 1986)

*Pis'ma Zh. Eksp. Teor. Fiz.* **44**, No. 11, 526–529 (10 December 1986)

A new method is proposed for reconstructing the energy spectrum of the electrons at an inversion layer from measurements of the quantum oscillations of the charge in a silicon metal-oxide-semiconductor structure in a quantizing magnetic field.

The method has been tested experimentally. In the reconstruction of the spectrum, the only parameter which is taken from other experiments is the effective electron mass  $m^*$ .

Electron inversion layers at the silicon (001) surface in a quantizing magnetic field have been under study for more than twenty years now, but the electron energy spectrum there has yet to be finally determined. The only results which can be regarded as solidly established are the values of the effective masses (Ref. 1, for example) and of the energy splittings between the Landau sublevels in cases in which the Fermi level  $\epsilon_F$  lies halfway across the corresponding energy gap.<sup>2</sup> In all other cases, the values of the spin splitting  $\Delta_s$  and the valley splitting  $\Delta_v$ , the distributions of the energy state density  $D$  in the energy at the Landau level, and their dependence on  $\epsilon_F$  remain undetermined, despite the many studies in this field (reviewed in Refs. 3–5). The results of most of the experiments cannot be accepted as reliable because of the use of many assumptions of a model nature.

The energy splittings in the spectrum were determined in Ref. 2 without any model assumptions from an analysis of the shape of the oscillations in the Fermi energy<sup>1)</sup> of electrons at a two-dimensional ( $2D$ ) layer,  $\epsilon_F$ , as a function of  $H$ . Again in this case, however, it was not possible to directly determine, for example, how the

energies of the Landau sublevels  $\epsilon_i$  ( $i$  is an integer) depend on the filling of these levels, since it did not appear possible to separate the dependence of  $\epsilon_F$  on the level filling  $\nu = n_s/n_H$  ( $n_H = eH/2\pi\hbar c$  is the degree of degeneracy of the level, and  $n_s$  is the density of electrons at the inversion layer), from the dependence on  $\epsilon_i$ , which also depends on  $H$ , as  $H$  was varied.

It is clear that the oscillations of  $\epsilon_F(H)_{n_s = \text{const}}$ , studied in Ref. 2, should be accompanied by corresponding oscillations on the curve of  $\epsilon_F(n_s)_{H = \text{const}}$ . It has proved possible to study these oscillations by measuring the dependence of  $n_s(V_g)_H$ . Oscillations of the charge of the inversion layer,  $Q_s = eSn_s$  [ $S$  is the area of the metal-oxide-semiconductor (MOS) structure, and  $e$  is the elementary charge], were first discovered in Ref. 6. An increase in the sensitivity of the measurement apparatus by a factor of  $10^3$  made it possible to study these oscillations over a broad field range and for various indices of the energy sublevels. The first results of these experiments, which we are reporting to the present letter, demonstrate that it is possible to reconstruct the spectrum and its dependence on the electron density.

### EXPERIMENTAL PROCEDURE

In the experiments (the layout is shown schematically in the inset in Fig. 1), we measure the current ( $J_g$ ) charging the MOS structure as the gate voltage  $V_g$  is varied linearly over time. The reason for the difficulty in measuring the oscillations of  $\delta J_g(V_g)$  is that they are quite small against the background of the large direct charging current  $\langle J_g \rangle = C_0(\partial V_g/\partial t) \sim 10^{-9}$  A, where  $C_0 = 700$  pF is the capacitance of the MOS structure. In our experiments, for example,  $\delta J_g/J_g$  is  $\lesssim 1\%$ . Furthermore, it proved possible to study the oscillations  $\delta J_g$  only at  $T \gg 1$  K, since at lower values of  $T$  time-varying surges in the current corresponding to a recharging of the inversion layer

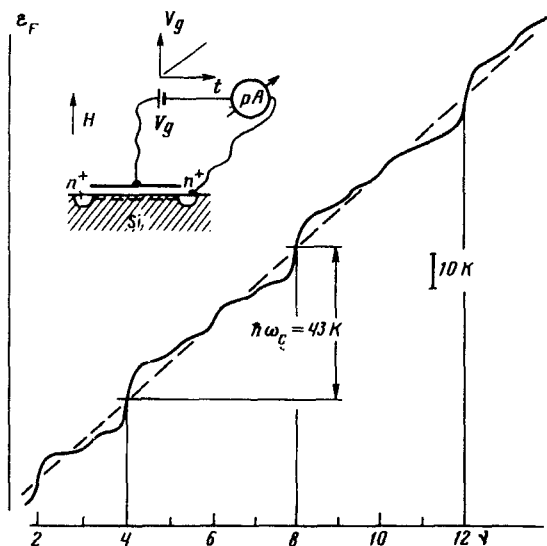


FIG. 1. Fermi energy versus the electron density (in units of  $n_H = 1.7 \times 10^{11} \text{ cm}^{-2}$ ) at  $H = 70$  kOe and  $T = 1.3$  K. The inset shows the measurement layout.

appear and grow sharply; these surges are considerably greater in magnitude. The test samples are those studied in Ref. 7.

## JUSTIFICATION OF THE METHOD

The voltage  $V_g$  specifies the difference between the electrochemical potentials of the inversion layer and of the gate:  $V_g = (\mu_{0g} - \mu_0 + e\Delta\varphi)/e$ , where  $\mu_0 = \epsilon_F + E_0$  is the chemical potential of the 2D layer,  $\mu_{0g}$  is the chemical potential of the gate, and  $\Delta\varphi$  is the difference between the electric potentials of the gate and the layer. The charge on the MOS structure is  $Q_s = C_0\Delta\varphi$ , where  $C_0$  is the capacitance of the capacitor formed by the gate and the 2D layer. The measured charging current is

$$J_g = dQ_s/dt = (dQ_s/dV_g)(dV_g/dt),$$

and

$$(dQ_s/dV_g) = (dC_0/dV_g)\{V_g + (\mu_0 - \mu_{0g})/e\} + C_0 \left[ 1 - \frac{d(\mu_0 - \mu_{0g})}{edV_g} \right].$$

We see that an important role is played in this expression by  $(\mu_0 - \mu_{0g})/e$ : the contact potential difference between the gate and the layer, which we studied in Ref. 2.

The second term in brackets, which contains information on the spectrum, is much smaller than 1, so that we distinguished  $d\epsilon_F/dV_g$  by carrying our measurements in succession at  $H \neq 0$  and  $H = 0$  and subtracting the second results from the first. Since neither the energy ( $E_0$ ) of the bottom of the electrical quantization band<sup>3</sup> nor the average distance from the electrons to the surface<sup>3</sup> (and thus  $C_0$ ) should depend on  $H$ , this difference is

$$\begin{aligned} \Delta(dQ_s/dV_g) &\equiv (dQ_s/dV_g)_{H \neq 0} - (dQ_s/dV_g)_{H=0} \\ &= \frac{1}{e} \{ (dC_0/dV_g)(\epsilon_F|_{H \neq 0} - \epsilon_F|_{H=0}) \} + \frac{C_0}{e} [(d\epsilon_F/dV_g)_{H \neq 0} - (d\epsilon_F/dV_g)_{H=0}]. \end{aligned}$$

Since  $dC_0/dV_g$  is small ( $\sim 10^{-13}$  F/V), the first terms of this sum (in braces) is smaller than the second by three or four orders of magnitude and can be ignored. Furthermore, by virtue of the relation  $(d\mu_0/dV_g)/e \ll 1$  we can replace  $V_g$  by  $Sn_s e/C_0$ . Experimentally we thus determine the dependence

$$\Delta(dQ_s/dV_g) = \frac{C_0^2}{e^2 S} (\partial\epsilon_F/\partial n_s|_{H \neq 0} - \partial\epsilon_F/\partial n_s|_{H=0}) \text{ on } n_s.$$

Integrating the result, we find the dependence  $\epsilon_F(n_s)_{H=\text{const}}$ . The only parameter here—the value of  $\partial\epsilon_F/\partial n_s|_{H=0}$ —can easily be determined from the known value of the effective mass of an electron. For  $m^* = 0.21m_e^1$  we have  $\partial\epsilon_F/\partial n_s = 6.6 \times 10^{-11} \text{K} \cdot \text{cm}^2$ .

## EXPERIMENTAL RESULTS

Figure 1 shows the measured dependence  $\epsilon_F^-(v)$ . We can clearly distinguish steep regions, where the Fermi level undergoes an abrupt transition from one Landau sub-

$(D^*)^{-1} \cdot 10^{10}, \text{cm}^2 \cdot \text{K}$

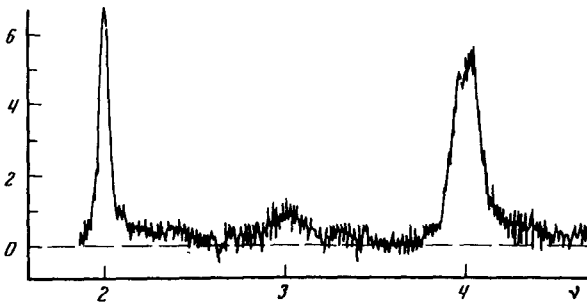


FIG. 2. The reciprocal "state density"  $(D^*)^{-1}$  versus the electron density.  $H = 70$  kOe,  $T = 1.9$  K.

level to another (near integer values of  $\nu$ ). We can see transits even at odd values of  $\nu$ , corresponding to valley splitting  $\Delta_\nu$  of smaller magnitude. The distances between the sublevels near integer values of  $\nu$ , found from this curve by a procedure like that of Ref. 2, agree with the results of Ref. 2.

*State density.* The slope of the curve in Fig. 1, i.e.,  $\partial \epsilon_F / \partial n_s$ , increases by an order of magnitude near integer values of  $\nu$ , implying a significant decrease in the state density  $D(\epsilon)$  in this region, i.e., a slight overlap of levels:  $D_{\min} \sim 0.1 D_{\max}$ . Figure 2 shows the derivative  $(\partial \epsilon_F / \partial n_s)_{H = \text{const}} \equiv (D^*)^{-1}$  of the curve in Fig. 1; this derivative is proportional to the difference between the gate charging currents  $J_g$  measured in the presence and absence of the field; see (2). This quantity is an indirect measure of the state density, since we have  $D^* \rightarrow D(\epsilon)$  in the one-electron approximation in the limit  $T \rightarrow 0$ . Near integer values  $\nu \approx i$ , the quantity  $(\partial \epsilon_F / \partial n_s)$  depends on (first) the tem-

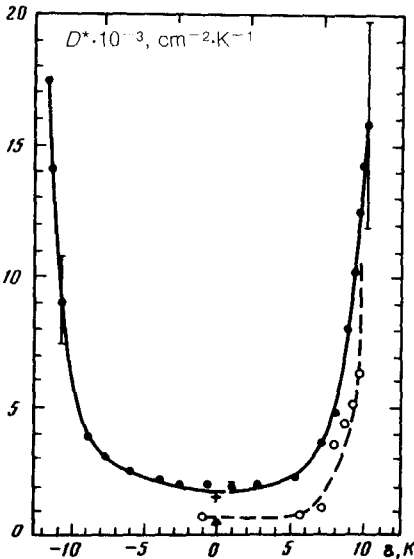


FIG. 3. The "state density"  $D^*$  near  $\nu = 4$  at  $H = 70$  kOe.  $\bullet$ — $T = 1.9$  K;  $\circ$ — $T = 1.3$  K;  $\blacktriangle$ —calculated from the data of Ref. 2 for  $T = 1.3$  K,  $H = 80$  kOe, and  $\nu = 8$ ;  $+$ —calculated from the data of Refs. 7 and 9 for  $T = 2$  K,  $H = 70$  kOe, and  $\nu = 4$ .

perature, (second) the corresponding energy splitting  $\Delta$  (in particular, on  $H$ ), and (third) the index of the energy gap,  $i$ . All three curves are monotonic;  $\partial\epsilon_F/\partial n_s$  decreases with increasing  $T$  or  $i$  and with decreasing  $\Delta$ . Since the relationship between  $\epsilon_F$  and  $\nu$  is known from Fig. 1, we can easily reconstruct the dependence of  $(\partial n_s/\partial\epsilon_F)$  on  $\epsilon_F$  (Fig. 3).

The values of  $(\partial n_s/\partial\epsilon_F)$  at  $\nu \approx i$  determined from these experiments agree well with the measurements of Ref. 2 and also with results calculated from measurements of the current distribution<sup>7</sup> (according to Ref. 9, the current distribution is also related to the quantity  $\partial\epsilon_F/\partial n_s$ ) (Fig. 3).

*Splittings in the energy spectrum.* From Fig. 1 we can determine the splittings not only at integer values of  $\nu$  but also at other values, e.g., half-integer values. In particular, it turns out that for a half-filling of the levels of this sort the valley splittings are (first) independent of  $n_s$  (as in the case of integer values of  $\nu$ ; Ref. 8) and (second) slightly smaller than the splittings at integer values of  $\nu$ . The values found here,  $\Delta_{3 \pm 1/2} = 6$ ,  $\Delta_{5 \pm 1/2} = 7.5$ ,  $\Delta_{7 \pm 1/2} = 5.9$ , and  $\Delta_{9 \pm 1/2} = 7.2$  K, are smaller than  $\Delta_3 = 8$  K. This result is a direct indication that the energies of the sublevels depend on their populations, and it confirms the dependence of  $\Delta_\nu$  on the value population predicted theoretically (Secs. VI and VII in Ref. 3).

We hope that future experiments of this type will yield a comprehensive picture of the electron energy spectrum of an inversion layer in a quantizing magnetic field.

We wish to thank N. S. Ivanov and A. K. Yanysh for technical assistance.

<sup>1</sup>We are placing the origin of the scale of the Fermi energy  $\epsilon_F$  at the bottom of the electrical-quantization band,  $E_0$  (Ref. 4).

<sup>1</sup>U. Kunze and G. Lutz, Surf. Sci. **142**, 314 (1984).

<sup>2</sup>V. M. Pudalov, S. G. Semenchinskiĭ, and V. S. Édel'man, Zh. Eksp. Teor. Fiz. **89**, 1870 (1985) [Sov. Phys. JETP **62**, 1079 (1985)].

<sup>3</sup>T. Ando, A. B. Fowler, and F. Stern, Rev. Mod. Phys. **54**, 437 (1982).

<sup>4</sup>V. M. Pudalov and S. G. Semenchinskiĭ, Poverkhnost' **4**, 5 (1984).

<sup>5</sup>É. I. Rashba and V. B. Timofeev, Fiz. Tekh. Poluprovodn., 1986.

<sup>6</sup>V. M. Pudalov, S. G. Semenchinsky, and V. S. Edel'man, Solid State Commun. **51**, 713 (1984).

<sup>7</sup>S. G. Semenchinskiĭ, Zh. Eksp. Teor. Fiz. **91**, 1804 (1986) [Sov. Phys. JETP (to be published)].

<sup>8</sup>V. M. Pudalov, S. G. Semenchinskiĭ, and V. S. Édel'man, Pis'ma Zh. Eksp. Teor. Fiz. **41**, 265 (1985) [JETP Lett. **41**, 325 (1985)].

<sup>9</sup>V. M. Pudalov and S. G. Semenchinskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **42**, 188 (1985) [JETP Lett. **42**, 228 (1985)].

Translated by Dave Parsons