Explicitly broken $N = 4$ supersymmetry model with a finite vacuum energy

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The $N = 4$ supersymmetry model with an explicit soft breaking of the supersymmetry is shown to have a finite vacuum energy density in any order of perturbation theory if the mass terms satisfy the additional sum rule $\Sigma_{J=0,1/2}(2J+1)(-1)^{2J}m_J^4 = 0$.

It has recently been shown that the $N = 4$ supersymmetry model[1] is ultraviolet-finite in all orders of perturbation theory.[2] Furthermore, the addition of mass terms, which explicitly break the $N = 4$ supersymmetry, preserves the property of finiteness of the Green's functions (in a special gauge).[3,4]

In the present letter we show that the addition of mass terms, which explicitly break the $N = 4$ supersymmetry, leads to a finite vacuum energy density if the mass terms satisfy the additional sum rule

$$\sum_{J=0,\,1/2} (2J+1)\,(-1)^{2J} m_J^4 = 0. \qquad (1)$$

We rewrite the Lagrangian of the $N = 4$ supersymmetry model as[1]

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + i\bar{\Psi}^\alpha \nabla \Psi_\alpha + \frac{1}{4} \nabla_\mu \bar{H}^{\alpha\beta} \nabla_\mu H_{\alpha\beta} - \frac{g}{\sqrt{2}} (\bar{H}^{\alpha\beta} \Psi_\alpha \times \Psi_\beta + \text{H.a.})$$

$$-\frac{g^2}{16} \bar{H}^{\alpha\beta} \times H^{\gamma\delta} H_{\alpha\beta} \times H_{\gamma\delta},$$

$$\nabla_\mu = \partial_\mu + gA_\mu \times$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + gA_\mu \times A_\nu, \qquad (2)$$

where the vector field A_μ, the chiral spinors Ψ_α, and the scalars $H_{\alpha\beta}$ transform under an associated representation of the gauge group. The indices α and β take on the values 1,...,4, which correspond to the global SU(4) symmetry, with respect to which A_μ is a singlet, Ψ_α is a quartet, and $H_{\alpha\beta}$ is a real sextet:

$$H_{\alpha\beta} = \frac{1}{2} \epsilon_{\alpha\beta\gamma\delta} \bar{H}^{\gamma\delta},$$

$$\bar{H}^{\gamma\delta} = (H_{\gamma\delta})^*.$$

As was shown in Refs. 3 and 4, the addition of mass terms, which explicitly break the supersymmetry, does not disrupt the property of finiteness of the Green's functions (in a special gauge), provided that the mass terms satisfy the sum rule

$$\sum_{J=0,1/2} (2J+1)(-1)^{2J} m_J^2 = 0. \tag{3}$$

Let us consider the mass terms[3,4]

$$\Delta \mathcal{L} = -m_i^2 \Phi_i^+ \Phi_i - m_{ij}^2 \Phi_i \Phi_j - m_{ij}^2 \Phi_i^+ \Phi_j^+ - \frac{1}{2} M_\alpha (\Psi_\alpha \Psi_\alpha + \text{H.a.}) - \sqrt{2} g (M_1 \Phi_1 \cdot \Phi_3^+ x \Phi_2^+ + M_2 \Phi_1^+ \cdot \Phi_3^+ \times \Phi_2 + M_3 \Phi_1^+ \Phi_3 \times \Phi_2^+ + M_4 \Phi_1 \cdot \Phi_3 \times \Phi_2 + \text{H.a.}) \equiv - m_i^2 \Phi_i^+ \Phi_i - m_{ij}^2 (\Phi_i \Phi_j + \Phi_i^+ \Phi_j^+) - M_\alpha (K_\alpha + \text{H.a.}). \tag{4}$$

We note that mass term (4), with $m_i = M_i$ $(i = 1,2,3)$ and

$$m_{ij} = M_4 = 0, \tag{5}$$

preserves the $N = 1$ supersymmetry. The fields A_μ, Ψ_i, Φ_i, and Ψ_4 are grouped into supermultiplets:

(Ψ_4, A_μ), a vector multiplet,

(Φ_i, Ψ_i), chiral multiplets.

As a result of the existence of the $N = 1$ supersymmetry, the energy of the vacuum is zero in case (5).

We expand the expression for the vacuum energy density in a series in the mass parameters m_i, m_{ij}, and M_j, retaining terms of up to fourth order[1)]:

$$\Delta E = m_i^2 c_i^1 + M_j^2 c_j^2 + m_{ij}^2 m_{ij}^2 c_{ij}^7 + m_i^2 m_j^2 c_{ij}^3 + M_i^2 M_j^2 c_{ij}^4 + m_i^2 M_j^2 c_{ij}^5 + m_{ij}^2 M_k M_l c_{ijkl}^6 + O(m^6). \tag{6}$$

According to the standard rules for calculations on divergences of Feynman diagrams,[5] the first two coefficients, c_i^1 and c_j^2, are quadratically divergent, while the coefficients c_{ij}^3, c_{ij}^4, c_{ij}^5, c_{ijkl}^6, and c_{ij}^7 are logarithmically divergent. To prove that the vacuum energy density is finite, we must therefore verify that all the terms in the expansion of ΔE in the mass parameters of up to fourth order inclusively cancel out.

In case (5), the mass terms do not break the $N = 1$ supersymmetry, so that the vacuum energy is zero in each order in m_i. We thus have the sum rules

$$c_i^1 + c_i^2 = 0, \tag{7}$$

$$c_{ij}^3 + c_{ij}^4 + c_{ij}^5 = 0, \tag{8}$$

$i, j = 1, 2, 3.$

The SU(4) symmetry of Lagrangian (2) leads to the relations

$$c_i^1 = c_j^1, \quad c_i^2 = c_j^2, \tag{9}$$

$$c_{ij}^k = c_0^k + \delta_{ij} c_1^k, \tag{10}$$

$$k = 3, 4, 5, 7.$$

Using the result of an analysis of the structure of the ultraviolet divergences in $N = 1$ supersymmetry theories with a soft breaking of the supersymmetry,[6] we find

$$c_{ijkl}^6 = 0 .$$

The first two coefficients in expansion (6) vanish by virtue of relations (7) and (9) and sum rule (3). Using relations (8) and (10) and sum rules (1) and (3), we find

$$\Delta E = \sum_{i=1}^{3} (m_i^2 M_i^2 - m_i^4) c_1^5 + O(m^6) . \tag{11}$$

The coefficient c_{ij}^5 is

$$c_{ij}^5 = \int d^4x d^4y < 0 | T(\Phi_i^+(0)\,\Phi_i(0),\ K_j(x),\ K_j^+(y)) >_{con}^{N=4} . \tag{12}$$

Since $K_j(x)$ transforms as a component of a 10-plet under the SU(4) global group, while $\Phi_i^+(x)\Phi_i(x)$ transforms as $1 + 20_s$, we find

$$c_{ij}^5 = c_0^5 , \quad c_1^5 = 0. \tag{13}$$

We have thus proved that the energy density of the vacuum is finite.

The $N = 4$ supersymmetry model with a soft breaking of supersymmetry is therefore the first example of a nontrivial four-dimensional theory with a finite vacuum energy density. In the single-loop approximation, the vacuum energy density is

$$\Delta E = \frac{1}{64\pi^2} \sum_{j=0,\,1/2} (2J+1)\, m_J^4 \, (-1)^{2J} \ln \frac{m_J^2}{\mu^2} . \tag{14}$$

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[1)]In deriving expansion (6), we leaned heavily on the invariance of Lagrangian (2) under the transformations $\Psi_j \to \exp(i\alpha_j)\Psi_j, \Psi_4 \to \exp(-i\Sigma_j \alpha_j)\Psi_4, \Phi_k \to \exp(i\alpha_k - i\Sigma_j \alpha_j)\Phi_k$.

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