Condensates and the low-energy region in quantum chromodynamics

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The properties of the low-energy region in quantum chromodynamics are determined completely by gluon and quark condensates. The condition for the existence of the region is that the gluon condensate must be positive.

The governing role played by the gluon and quark condensates $\langle G^2_{\mu\nu}\rangle$ and $\langle\bar{\psi}\psi\rangle$ in low energy phenomena in quantum chromodynamics (QCD) has been established well by the sum rules.[1] So far, however, we do not know the relationship between condensates and the properties of the low-energy region in QCD, where processes involving a breaking of chiral symmetry are important. In the present letter we show that it is possible to introduce properties of the low-energy region which are determined unambiguously by condensates.

We consider a generating functional for the vacuum expectation values of the quark currents, which also depends on the external fields V_μ and A_μ:

$$Z(V,A) = \int (DG) Z_\psi (V,A,G) \exp iW_{YM}, \tag{1}$$

$$Z_\psi (V,A,G) = \int D\bar{\psi} D\psi \exp\left(i\int \bar{\psi}\bar{\not{D}}\psi d^4x\right), \tag{2}$$

where G_μ is the gluon field, W_{YM} is the action of the gluon field (including Faddeev-Popov ghosts), and $V_\mu = V_{\mu a} T_a$, where T_a are anti-Hermitian generators of the group of color and flavors. The total Dirac operator $\not{D}$ for quarks is given by $\not{D} = i\gamma^\mu [\partial_\mu + (V_\mu + \gamma_5 A_\mu)\otimes 1_c + g1_f \otimes G_\mu]$, where 1_c and 1_f are unit matrices in the space of color and flavor.

In fermion integral (2) we single out the relativistically invariant low-energy region L of the integration over quarks, which satisfies the following conditions: (a) The physical region L is determined by the circumstance that nonperturbative, chiral-noninvariant fluctuations of quarks characterized by a condensate $\langle\bar{\psi}\psi\rangle$ are dominant in it; (b) the region L is gauge-invariant, and the vector isospin currents are conserved; (c) the region L is stable with respect to fluctuations of the magnitude of the quark condensate. We go over into Euclidean space in (1) and (2), and we consider the eigenvalues of K of the total Dirac operator: $\not{D}\psi_{K\alpha} = K\psi_{K\alpha}$, where α is the index specifying the polarization and internal degrees of freedom. We introduce two parameters with the dimensionality of a mass, Λ and M, and we define the region L by the relation

$$-\Lambda + M \leqslant K \leqslant \Lambda + M, \quad 0 \leqslant M \leqslant \Lambda. \tag{3}$$

Both Λ and M are invariant under vector gauge and local chiral transformations of both the flavor group SU(2) and the color group. In this approximation, the condensate is $\langle\bar{\psi}\psi\rangle \sim 1_f \otimes 1_c$.

In the presence of external fields, the quark condensate is equal to the vacuum value of the quark density, averaged over the volume. In Euclidean space, for massless quarks, we have

$$\langle \bar{\psi}\psi\rangle_E = - \int d^4K \frac{2N_c}{K} \{\theta(\Lambda + M - K) - \theta(\Lambda - M - K)\} = \frac{N_c}{2\pi^2}\left(\Lambda^2 M + \frac{M^3}{3}\right), \quad (4)$$

where we have made use of the eigenfunction basis of the total Dirac operator. In Minkowski space $(M \to -iM)$ we have

$$\langle \bar{\psi}\psi\rangle = -\frac{N_c}{2\pi^2}\left(\Lambda^2 M - \frac{M^3}{3}\right), \quad (5)$$

where N_c is the number of colors.

The size of the quark condensate varies under quark fluctuations $\psi(x) \to \exp[-\sigma(x)]\psi(x)$ with a scalar field $\sigma = \sigma_0 + \tau^a\sigma_a$, which is generally realized in both singlet and associated representations of the flavor group. If the region L is to be stable, such fluctuations must be suppressed. Whether they can be suppressed is determined by the effective action $W_{\text{eff}}(\sigma)$ for the field σ, which is generated by a conformal anomaly. Here the gluons play a governing role. In calculating $W_{\text{eff}}(\sigma)$ we accordingly assume $V_\mu = A_\mu = 0$, so that we have $\not{D} = i(\not{\partial} + g\not{G})$. In addition to $\not{D}$, we need to also consider the conformally transformed Dirac operator $\not{D}_\sigma = e^\sigma \not{D} e^\sigma$. We denote the corresponding fermion integrals in (2) by $Z_\psi(G,1)$ and $Z_\psi(G,\sigma)$. The effective action $W_{\text{eff}}(\sigma)$ is then given by the following expressions (in Euclidean space):

$$\exp(-W_{\text{eff}}(\sigma)) \equiv Z_\psi(G,1) Z_\psi^{-1}(G,\sigma), \quad (6)$$

$$W_{\text{eff}}(\sigma) = \int_0^1 ds \int d^4x \, 2\,\text{tr}\{\sigma(x)\langle x | \theta(\Lambda^2 - (\not{D}_{s\sigma} - M)^2) | x\rangle\}. \quad (7)$$

In deriving (7), we made use of a finite-mode regularization.[2] Expression (7) was found through a conformal anomaly integration.

To determine the conditions for the stability of the region L under a variation of the quark condensate, it is sufficient to consider static fields $\sigma = \sigma_c = \text{const}$ and to examine the effective potential $V(\sigma_c)$. Evaluating (7) in Euclidean space, and then transforming to Minkowski space, we find the following expression for the effective potential:

$$V(\sigma_c) = \frac{N_f}{16\pi^2}\left\{\frac{N_c}{4}(e^{-8\sigma_c} - 1)(6\Lambda^2M^2 - \Lambda^4 - M^4) + (g^2\sigma_c/3)\sum_a (G^a_{\mu\nu})^2\right\}, \quad (8)$$

which contains a linear gluon invariant. Here N_f is the number of flavors. Taking an average over the gluon fields in our basic functional (1) at *large* values of N_c leads to a condensate $\langle G_{\mu\nu}^2 \rangle$ as the leading term for $G_{\mu\nu}^2$ in (8). Low-energy region (3) is stable with respect to fluctuations of the condensate $\langle \bar{\psi}\psi \rangle$ if the effective potential has a minimum at $\sigma_c = 0$, i.e., in the absence of fluctuations. The condition for an extremum of the effective potential generates a relationship between the parameters Λ and M and the gluon condensate:

$$6N_c (6\Lambda^2 M^2 - \Lambda^4 - M^4) = \langle g^2 \sum_a (G_{\mu\nu}^a)^2 \rangle . \tag{9}$$

The extremum of the effective potential in the case $\sigma_c = 0$ is a minimum if the gluon condensate is positive:

$$\langle \Sigma (G_{\mu\nu}^a)^2 \rangle > 0 . \tag{10}$$

We thus see that we cannot speak in terms of a low-energy region L in QCD if the gluon condensate is negative or zero. Expressions (5) and (9) describe the quark and gluon condensates in terms of the parameters Λ and M of the low-energy region. This region is thus determined unambiguously by the condensates.

The parameters Λ and M (the asymmetry of the spectrum) also determine the seed constant for pion decay in the chiral Lagrangian[3]: $F_0^2 = (N_c/4\pi^2)(\Lambda^2 - M^2)$. Taking the chiral logarithms into account, we find that we would need $F_0 = 88$ MeV for $F_\pi = 93$ MeV.

Choosing the parameter values $\Lambda = 450$ MeV and $M = 320$ MeV, we find, with $F_0 = 88$ MeV and $\langle \bar{\psi}\psi \rangle = -(200 \text{ MeV})^3$, a completely reasonable value for the gluon condensate: $(g^2/4\pi^2)\langle \Sigma (G_{\mu\nu}^a)^2 \rangle = (415 \text{ MeV})^4$.

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