Study of particles trapped by a magnetic field

G. M. Zaslavskiĭ, S. S. Moiseev, R. Z. Sagdeev, and A. A. Chernikov
Institute of Space Research, Academy of Sciences of the USSR

(Submitted 13 November 1985)
Pis'ma Zh. Eksp. Teor. Fiz. **43**, No. 1, 18–21 (10 January 1986)

A new type of radiation which occurs when particles are accelerated in the field of a longitudinal wave and in a transverse magnetic field is studied. The characteristics of spontaneous radiation are obtained. The influence of collective effects on the radiation is analyzed.

1. Particles that move in front of a wave front and parallel to it in a magnetic field are accelerated.[1] In turning the particles, the magnetic field causes them to undergo multiple collisions with the wave. Each collision of this sort increases the velocity of the particles. If the particles are trapped by a potential wave $\varphi(x,t) = \varphi_0 \cos(kx - \omega t)$, moving across a magnetic field $B_0 \| 0z$, then the same acceleration mechanism causes the trapped particles in the nonrelativistic case to spill out from the potential well of the wave.[2] In the relativistic case, the velocity of particles along the wave front is limited by the velocity of light, so that if the field $E_0 = k\varphi_0$ is higher than B_0, the particles may become trapped in the potential well and the trapped particles will be accelerated along the y axis.[3,4]

In this letter we predict theoretically a new type of radiation that can occur when electrons move in this manner. The radiation is caused by oscillations of the trapped

 0021-3640/86/010021-04$01.00

particles in the direction perpendicular to the wave front as they move at a relativistic velocity along the wave front.

2. A system of canonical equations which describes the interaction of an electron with a potential wave in a transverse magnetic field can be written

$$\dot{x} = \frac{c^2 p_x}{\sqrt{m^2 c^4 + c^2 p_x^2 + e^2 B_0^2 x^2}}, \quad \dot{p}_x = -\frac{e^2 B_0^2 x}{\sqrt{m^2 c^4 + c^2 p_x^2 + e^2 B_0^2 x^2}} - eE_0 \sin(kx - \omega t). \tag{1}$$

In the case of a particle trapped by a wave field $v_x \cong \omega/k$, it can be inferred from (1), according to Refs. 3 and 4, that the momentum component along the wave front increases with time in an approximately linear manner:

$$p_y/mc \cong -\omega_H \beta_{wf} t , \tag{2}$$

where $\omega_H = eB_0/mc$ is the nonrelativistic cyclotron frequency, and $\beta_{wf} = \omega/kc$. The total energy of the particle, $\mathscr{E}$, increases with time as

$$\frac{\mathscr{E}}{mc^2} = \gamma(t) = \gamma_{wf} \sqrt{1 + \omega_H^2 \beta_{wf}^2 t^2}, \quad \gamma_{wf} = \frac{1}{\sqrt{1 - \beta_{wf}^2}}. \tag{3}$$

The small corrections to the transient ground state in (2) and (3) are described, according to (1), by the equation

$$\frac{d}{dt}\left\{\gamma(t)\frac{d\xi}{dt} + \frac{\beta_{wf}^2 \omega_H^2 t}{\gamma(t)} \xi \right\} + \frac{eE_0}{m\gamma_{wf}^2} \sin k\xi = -\frac{\beta_{wf}\omega_H^2 ct}{\gamma(t)}, \tag{4}$$

where $\xi(t) = x(t) - (\omega/k)t$. Under adiabatic conditions $\Omega_b = (eE_0 k/m)^{1/2} \gg \gamma_{wf}^2 \beta_{wf}\omega_H/\gamma^{1/2}$ and $E_0 \gg \gamma_{wf} B_0$ an approximate solution of the linearized equation (4) is

$$\xi(t) = -\frac{c\omega_H^2 \beta_{wf} \gamma_{wf}^2 t}{\Omega_b^2 \gamma(t)} + \frac{v_0 \gamma_{wf}^{3/2}}{\Omega_b} \left(\frac{\gamma_{wf}}{\gamma(t)}\right)^{\frac{1}{4} + \frac{1}{2\gamma_{wf}^2}} \sin \int_0^t dt' \frac{\Omega_b}{\gamma_{wf}\gamma^{1/2}(t')}, \tag{5}$$

where v_0 is the initial oscillation rate of the trapped particles. According to (5), the frequency Ω and the oscillation rate $\tilde{v}$ decrease over time as

$$\Omega = \Omega_b/\gamma_{wf}\gamma^{1/2}, \quad \tilde{v} = v_0 \left(\frac{\gamma_{\Phi}}{\gamma}\right)^{\frac{3}{4} + \frac{1}{2\gamma_{wf}^2}}. \tag{6}$$

3. To calculate the spontaneous radiation of an electron which moves at a relativistic velocity $u \cong c$ and which oscillates along the x axis in accordance with (5), we can use some of the results of the wiggler radiation theory[5] (see also Ref. 6). In our case, however, both the oscillation frequency and the oscillation amplitude depend on the relativistic factor γ, i.e., the energy. It turns out that the radiation intensity I of a single electron depends on its energy in the following way:

$$I=\frac{1}{3}\left(\frac{v_0}{c}\right)^2 \frac{e^2\Omega_b^2\gamma^{3/2}}{c\gamma_{\rm wf}^{1/2}}\left(\frac{\gamma_{\rm wf}}{\gamma}\right)^{\frac{1}{\gamma_{\rm wf}^2}}. \quad (7)$$

The frequency of the emitted waves, $\tilde{\omega}$, depends on the angle θ between the direction of the velocity and the wave vector of the wave:

$$\tilde{\omega}=\frac{\Omega}{1-(u/c)\cos\theta}. \quad (8)$$

If the waves are emitted precisely along the beam ($\theta=0$), they will have a maximum frequency $\tilde{\omega}=\omega_m$, where

$$\omega_m=2\Omega_b\frac{\gamma^{3/2}}{\gamma_{\rm wf}}. \quad (9)$$

The radiation intensity is substantially different from zero only within a narrow cone of angles, $\theta \ll \theta_m \cong \gamma^{-1}$.

Let us estimate the radiation power of a single electron when it is accelerated to an energy $\mathscr{E}\cong 1$GeV. Setting $v_0/c\cong 0.1$, $\gamma_f=2$, and $\Omega_b\cong 10^{13}\ \mathrm{s}^{-1}$ in Eq. (7), we find $I\cong 3\times 10^{-8}$ W. The characteristic length of waves emitted in this case is $\lambda\sim 30$ Å.

4. The coherence effects must be taken into account in the case of sufficiently large beam density. A dispersion equation which describes a coherent interaction is similar to a corresponding equation of the theory of free-electron lasers.[7] In the case of a one-dimensional interaction of waves (emission along a beam with the excitation of the beam wave which propagates in the opposite direction), this equation, at $\gamma_f \gtrsim 1$, has the form

$$\left[(\omega-ku)^2-\frac{\omega_L^2}{\gamma^3}\right]\left[(\omega-\Omega)^2-c^2k^2-\frac{\omega_L^2}{\gamma}\right]=\frac{\omega_L^2}{\gamma^3}\frac{k^2\tilde{v}^2}{4}, \quad (10)$$

whre ω and k is the frequency and the wave number of the beam mode, and $\omega_L=(4\pi n_b e^2/m)^{1/2}$ is the Langmuir frequency of the beam ($\omega_L \gg \omega_H$). At resonance, from (10) we find relation (9) for the frequency of the generated electromagnetic wave. In the case of weak coupling of waves ($\delta \ll \omega_L\gamma^{-3/2}$), for the instability growth rate δ from Eq. (10) we accordingly find

$$\delta=\frac{1}{4}\frac{\tilde{v}}{c\gamma^{3/4}}\sqrt{\omega_L\tilde{\omega}_m}. \quad (11)$$

Let us estimate the frequency (9) and the growth rate (11) for the emission by an electron beam with an energy $\mathscr{E}\cong 20$ MeV and density $n_b\cong 10^{13}\ \mathrm{cm}^{-3}$. Setting $v_0/c\sim 0.1$ and $\Omega_b\cong 10^{13}\ \mathrm{s}^{-1}$, we find $\delta\cong 3\times 10^9\ \mathrm{s}^{-1}$, which corresponds to an amplification length $L=c/\delta\sim 10$ cm. The frequency of the generated emission in this case lies in the visible light range, $\omega_m\cong 5\times 10^{15}\ \mathrm{s}^{-1}$.

5. To maintain the emission regime at a nearly steady level, the energy must be transferred from the relativistic motion along the wave front to the transverse oscillation. Calculations have shown that modulation of a plane wave front $E_0(y) = E_0(1 + \alpha \sin k_\perp y)$ gives rise to a parametric amplification of oscillations which is most pronounced at $k_\perp \cong [(\Omega_b \times 2)/c]\gamma^{-1/2}$. A nearly steady regime is reached when the growth rate of the parametric resonance is comparable to the reciprocal of the time of the damping of the oscillation amplitude of electrons due to the radiation. We should point out that energy can be transferred from a relativistic longitudinal motion to electron oscillations because of the anomalous Doppler effect. This mechanism can impose a constraint on the acceleration regime.

We notice that the emission frequency can be increased significantly by directing the accelerated particles into the region of the upper hybrid resonance of an inhomogeneous plasma. In this case the bounce frequency Ω_0 will increase[8] in proportion to $(\omega/\nu)^{3/4}$ (ν is the collision frequency or any other factor that restricts the field in the resonant region $\omega \gg \nu$). It should also be kept in mind that the monochromatic nature of radiation improves in this case because of a strong decrease of the acceleration rate in the resonant region due to a decrease in the phase velocity of the wave. In other words, the particle acceleration region is comprised principally of the resonance "wings," while the resonant region is the region of hard radiation.

[1]R. Z. Sagdeev, Voprosy teorii plazmy (Reviews of Plasma Physics), Atomizdat, Moscow, 1964, Vol. 4, p. 20.
[2]R. Z. Sagdeev and V. D. Shapiro, Pis'ma Zh. Eksp. Teor. Fiz. **17**, 389 (1973) [JETP Lett. **17**, 279 (1973)].
[3]T. Katsouleas and J. M. Dawson, Phys. Rev. Lett. **51**, 392 (1983).
[4]B. É. Gribov, R. Z. Sagdeev, V. D. Shapiro, and V. I. Shevchenko, Pis'ma Zh. Eksp. Teor. Fiz. **42**, 54 (1985) [JETP Lett. **42**, 63 (1985)].
[5]L. D. Landau and E. M. Lifshitz, Teoriya polya (The Classical Theory of Fields), Fizmatgiz, 1960, Sec. 77 [Pergamon Press, Oxford (1962)].
[6]D. F. Alferov, Yu. A. Bashmakov, and E. T. Bessonov, FIAN 80, Nauka, 1975, p. 108.
[7]Generatory kogerentnogo izlycheniya na svobodnykh élektronakh (Free-Electron Lasers), ed. by A. A. Rukhadze, Mir, Moscow, 1983.
[8]S. S. Moiseev, V. V. Mukhin, V. E. Novikov, and R. Z. Sagdeev, Dokl. Akad. Nauk SSSR **285**, 346 (1985) [Sov. Phys. Doklady **285**, No. 11 (1985)].

Translated by S. J. Amoretty