Hopping-conductivity fluctuations in small samples

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Pronounced fluctuations in the resistance of small samples upon changes in the magnetic field and in the arrangement of impurities are predicted in the region of variable-range hopping. Numerical simulations are also carried out.

Umbach *et al.*[1] discovered fluctuations in the resistance of small samples of a disordered metal upon changes in a magnetic field. This effect has been reproduced in numerical simulations, and Stone[2] has offered a qualitative explanation. Al'tshuler[3] and Lee and Stone[4] show that the amplitude of the fluctuations in the reciprocal of the resistance of a sample is on the order of $e^2/\hbar$, and at absolute zero it does not depend on the dimensions of the sample. Similar fluctuations have been predicted upon changes in the Fermi level, an electric field, or the arrangement or scattering amplitudes of impurities.[4–6] In the present letter we wish to call attention to the circumstance that the resistance of a small sample on the insulator side of a metal-insulator transition in the region of variable-range hopping should also fluctuate upon a change in the magnetic field or some other factor. Fluctuations in the resistance in the variable-range-hopping region have already been observed upon a change in the Fermi level in small quasi-1D metal-insulator-semiconductor structure.[7] Lee[8] has explained these fluctuations on the basis that the resistance of the structure is determined by the largest of the many series-connected resistances in a Miller-Abrahams network; i.e., the probability for the most difficult hop. Upon a change in the Fermi level, there is a change in the identity of the hop that plays this role, and there is a fluctuation in the resistance. In the present paper we are dealing with fluctuations of a different nature, which stem from fluctuations in the probability for a single hop, e.g., the most difficult one, upon a change in the magnetic field. These fluctuations turn out to be on the order of the probability itself, so that they may be observed both in cases in which the resistance of the sample is determined by a single hop and in large samples, where, of course, there will be an averaging of the fluctuations. As a result, the experimental limitations on the length of a sample for the observation of fluctuations may be less stringent than in a metal.

The reason for the fluctuations in the hopping probability is that in the variable-range-hopping region the electrons hop between well-separated impurities with energies close to the Fermi level, being scattered "on the way" by a large number of other impurities in a cigar-shaped region of length r and transverse dimension $\sqrt{ra}$, where r is the length of the hop, and a is the localization radius of the wave functions. If there is a large number of impurities with a negative scattering amplitude, the hopping probability amplitude will be the sum of contributions with random signs from the various paths along the scattering impurities.[9] The imposition of a magnetic field **H** perpendicular to the hopping direction gives rise to a random phase factor $e^{i\varphi_k}$ in the term of index k, where φ_k is given in order of magnitude by $\varphi_s \equiv r\sqrt{ra}(eH/c\hbar)$. When φ_s reaches a value on the order of π, i.e., when the magnetic flux through a longitudinal cross section of the cigar, $r\sqrt{ra}$, becomes comparable to the flux quantum, $\Phi_0 = c\hbar/e$, there will be a substantial change in each term, and the entire sum will change in a random direction by an amount on the order of its own value. The characteristic "period" of the fluctuations along the H scale is thus

$$H_c = \frac{\pi c\hbar}{r^{3/2}a^{1/2}e} \,. \tag{1}$$

In the metallic phase, H_c has been estimated[2] from the condition that the flux through a sample area r^2 is on the order of Φ_0.

To test (1), we have numerically simulated the hopping of a spin-zero particle in the model of a binary alloy; the simulation is described in detail in Ref. 9. We studied the tunneling of an electron between sites 1 and 2 of a 25^3 cubic lattice; these sites are at the ends of its (1, 1, 1) body diagonal. We assumed a field **H** in the (1, −1, 0) direction. The Hamiltonian is

$$\mathcal{H} = \sum_i \epsilon_i a_i^+ a_i + \sum_{i\neq j} V_{ij} a_i^+ a_j \,, \tag{2}$$

where i, j specify the lattice sites, and a_i^+ is the creation operator at site i. For nearest neighbors, we would have $V_{ij} = V\exp\{i\varphi_{ij}\}$, where $\varphi_{ij} = (e/2\hbar c)\mathbf{H}[\mathbf{r}_i\mathbf{r}_j]$. In other cases, we would have $V_{ij} = 0$. The distribution function of the energy ϵ_i for $i \neq 1, 2$ is

$$g(\epsilon_i) = (1-x)\,\delta\,(\epsilon_i - W) + x\,\delta\left(\epsilon_i - \frac{W}{A}\right) \,, \tag{3}$$

where $W > 0$. We calculate the effective overlap integral I between sites 1 and 2, with approximately zero energies, for the situation deep in the insulator region: $W, |W/A| \gg |V|$.

Figure 1 shows $L(H) \equiv \ln|I(H)/I(0)|$ as a function of H for one realization of the energies ϵ_i for $A = -8$ and $x = 0.3$. The magnetic field is given in units of $\widetilde{H} = 2c\hbar/el^2$, where l is the lattice constant.

In the field region shown in Fig. 1, the expectation value over the realizations, $\langle L(H)\rangle$, is positive and essentially independent of H. Figure 2 shows the correlation function

$$f(\Delta H) = [\overline{L(H+\Delta H)\,L(H)} - \overline{L(H)}^2]^{1/2} = [\overline{\ln|I(H+\Delta H)|\,\ln|I(H)|} - \overline{\ln|I(H)|}^2]^{1/2}, \tag{4}$$

where the superior bar, in contrast with the angle brackets, means the expectation value of the field H in the region $0.05 \leqslant H/\widetilde{H} \leqslant 0.28$. We see that the fluctuation amplitude is $f(0) \cong 0.55$, and the value of H_c at which we have $f(H_c) = (1/2)f(0)$ is $0.01\widetilde{H}$. Since the length of the diagonal of a cube is $r = 24\sqrt{3}l$, and we have[9] $a = l/\sqrt{3}$, it is a simple matter to show that the values found for H_c agree with estimate (1). If, in

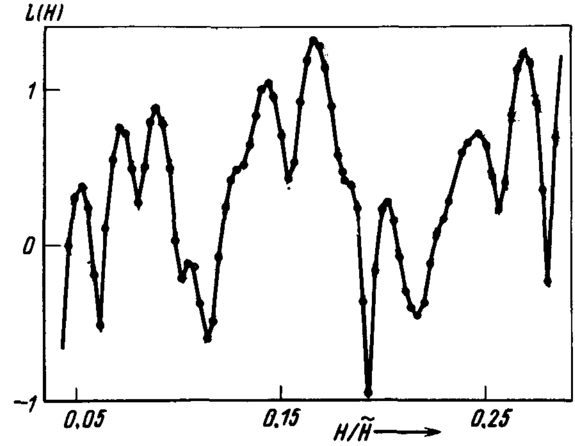

FIG. 1.

accordance with Ref. 8, we assume that the length a is on the order of 10^3 Å in the experimental situation of Ref. 7, while the length of the difficult hop is 10^4 Å, we find $H_c = 0.1$ T from (1); i.e., this field is not very strong. Consequently, a study of fluctuations in a magnetic field in experiments like that of Ref. 7 may be a convenient way to test Lee's explanation.[8]

It is interesting to compare the values of $f(0)$ with the fluctuations over realizations:

$$\delta(H) = [\langle \ln^2 |I(H)| \rangle - \langle \ln |I(H)| \rangle^2]^{1/2}. \tag{5}$$

(In contrast with Refs. 3–6, we are interested in the correlation functions $\ln|I|$, since the values of $\langle I \rangle^2$ in the insulator region are determined by rare realizations.)

For $A = -8$ and $x = 0.3$, the value of δ is 3.2 at $H = 0$ and 2.9 at $H > 0.01\tilde{H}$. The fluctuations over the realizations thus are much larger than the fluctuations in the magnetic field; i.e., the hypothesis of an ergodic situation, which Lee and Stone[4] formulated for metals, does not hold in the insulator region. The same pronounced dis-

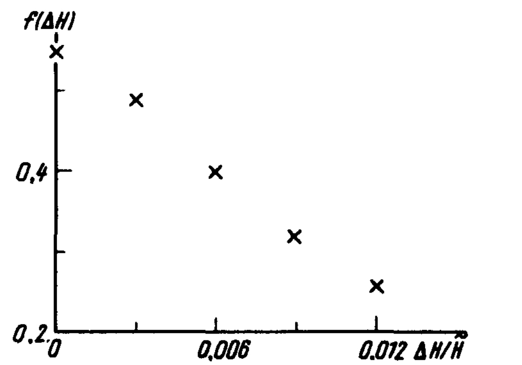

FIG. 2.

crepancy between f and δ was found in 100×100 2D arrays with $A = -1$ and $x = 0.5$.

We have also studied fluctuations of $\ln|I|$ caused by random changes in the coordinates in the cube of a fraction of the "scattering" sites with $\epsilon_i = W/A$. This calculation was carried out for $A = -8$ and $x = 0.3$. It was found that I changes by roughly a factor of two for the characteristic value $y = y_c = 0.05$, which corresponds to changes in the coordinates of about three sites with $\epsilon_i = W/A$ in the cigar. On the other hand, if $\ln|I|$ is to change by an amount on the order of the fluctuations in the realizations, $\delta \cong 3$, it is necessary to change the coordinates of nearly all the scattering sites.

A study of the fluctuations upon a change in y simulates the fluctuations in the conductivity due to the diffusion of impurities.[3]

We wish to stress that the phenomena described above occur only under the conditions $A < 0$ and $x > x_c$, where x_c is the point of the sign phase transition[9]; i.e., they occur only when the sign of I over large distances r is random.

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