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On a modification of the boundary state formalism in off-shell string theory

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We examine the application of boundary states in computing amplitudes in off-shell open string theory. We find a straightforward generalization of boundary state which produces the correct matrix elements with on-shell closed string states.

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1. Background independent string field theory [1 – 5] is an interesting approach to the problem of defining string theory off-shell. It has recently received a lot of attention, particularly as an approach to understanding the properties of unstable D-branes [6 – 8].

A concrete problem in the study of off-shell string theory in this formalism has been to understand its behavior in the a background tachyon field which is a quadratic function of the coordinates. This gives a tractable model where one can study phenomena such as tachyon condensation and D-brane-anti-D-brane annihilation [6 – 18]. In this context, the concept of boundary state, which describes the coupling of string world-sheets to a D-brane has been used by several authors [19 – 23].

The boundary state formalism could be useful in many circumstances: in computing D-brane tensions and cylinder amplitudes as well as in looking for the gravity counterparts of D-branes [24, 25]. This formalism was originally used to factorize open string amplitudes in terms of closed string states. This could be valuable in understanding the relationship between closed and open strings which is one of the central problems in uncovering the underlying symmetry of string theory. In the operator approach to string perturbation theory, the boundary state contains the coupling of closed strings to a D-brane.

In this Letter, we shall suggest a generalization of boundary states to be applied to the problem of computing off-shell amplitudes of the open Bosonic string. We consider the boundary state for a D-brane with a tachyon condensate and take the special case where the tachyon has a quadratic profile.

Then, we will examine the coupling of massless closed string states to the boundary state. There are two ways of analyzing this coupling. The first uses a sigma model approach. In that case, we insert the vertex operator for a graviton into the sigma model path integral with disc geometry and compute the expectation value. In the second approach, we construct a boundary state for the D-brane with tachyon condensate and consider the inner product of this state with the on-shell closed string graviton. We find that the result does not agree with the sigma model computation.

We then explain the reason for this disagreement and invent a modified boundary state which has the property that its inner products with all massless closed string states agree with amplitudes computed by inserting vertex operators for massless states into the sigma model path integral.

2. First, consider the sigma model which defines background independent string field theory. We will use the functional integral representation of the partition function of the Bosonic string. The world-sheet is the unit disc and the target space is 26-dimensional Euclidean space. The Bosonic string action is supplemented by a boundary term which contains the quadratic open string tachyon background:

$$Z(g,F,T_0,U) = (2\pi\alpha')^{13}\int \mathcal{D}X e^{-S(X,g,F,T_0,U)} = \\ = (2\pi\alpha')^{13}\int \mathcal{D}X \times \\ \times \exp\left\{-\frac{1}{2}\int_D d^2\sigma g_{\mu\nu}\partial^a X^\mu(\sigma)\partial^a X^\nu(\sigma) - \right. \\ -\oint_{\partial D} d\theta \left(\pi\alpha' F_{\mu\nu}X^\mu(\theta)\partial_\theta X^\nu(\theta) + \right. \\ \left.\left. +\frac{1}{2\pi}T_0 + \frac{\alpha'}{4}U_{\mu\nu}X^\mu(\theta)X^\nu(\theta)\right)\right\}, \qquad (1)$$

Here, α' is the inverse string tension. The world-sheet is a disc, D, for which we shall use complex coordinates

$z = e^{-\sigma_1 - i\sigma_2}$ with $0 < \sigma_1 < \infty$ and $0 \le \sigma_2 \le 2\pi$ is the parameterization of the disc D (or of the infinitely long half cylinder). We shall also sometimes use the coordinate $z = \rho e^{-i\theta}$. $0 \le \theta \le 2\pi$ is the parameterization of the boundary of the disc ∂D. $X^\mu(\sigma)$, $\mu = 1, ..., 26$ are maps of the string into the target space with the constant metric $g_{\mu\nu}$, $T(X) = T_0 + \frac{\pi\alpha'}{2} U_{\mu\nu} X^\mu X^\nu$ is the tachyon profile with the constant T_0 and constant matrix $U_{\mu\nu}$ of some rank and $F_{\mu\nu}$ is constant gauge field strength. This functional integral is taken with boundary conditions:

$$g_{\mu\nu}\, \partial_n X^\nu(\theta) + 2\pi\alpha' F_{\mu\nu} \partial_t X^\nu(\theta) + \frac{\alpha'}{2} U_{\mu\nu} X^\nu(\theta) = 0, \tag{2}$$

where ∂_n and ∂_t are the normal and tangential derivatives to the boundary ∂D. We use a non-standard normalization of X as in [26]. It is related to the standard one via the rescaling by $\sqrt{2\pi\alpha'}$.

The theory (1) is not conformaly invariant and represents a special example of the background independent string field theory [1 – 4]. Because of the conformal anomaly, this theory explicitly depends on the conformal factor of the world-sheet metric. The convention is to consider the theory on the unit disc with flat metric. The main advantage of the case of (1) is that the theory is gaussian and therefore is exactly solvable [1 – 3, 8 – 10]. For example, the renormalization group flow of the parameters $U_{\mu\nu}$ in the functional integral (1) describes the annihilation of a D25-brane in Bosonic string theory [8]. If the rank of the initial matrix $U_{\mu\nu}$ is $26 - p$, what is left after the annihilation of the D25-brane is a Dp-brane. This arises from the fact that the β-function for U is $\beta_U = -U$ and, hence, U flows to zero in the ultraviolet and to infinity in the infrared limits. Thus, we see from (2) that (if $F = 0$) the Neumann boundary conditions present in the UV limit where $U \sim 0$ evolve to Dirichlet boundary conditions on $26 - p$ coordinates, with the rest of the coordinates still obeying Newman boundary conditions. These final boundary conditions describe a Dp-brane.

The functional integral (1) is readily computed [1–3]:

$$Z(g, F, T_0, U) =$$

$$= (2\pi\alpha')^{13}\, e^{-T_0} \prod_{m=1}^{\infty} \frac{1}{\det\left(g + 2\pi\alpha' F + \dfrac{\alpha'}{2}\dfrac{U}{m}\right)} \times$$

$$\times \int dx_0\, e^{-\pi\alpha' \frac{U_{\mu\nu}}{2} x_0^\mu x_0^\nu} =$$

$$= (2\pi\alpha')^{13} \frac{1}{\sqrt{\pi\alpha' \det(U)}}\, e^{-T_0} \times$$

$$\times \prod_{m=1}^{\infty} \frac{1}{\det\left(g + 2\pi\alpha' F + \dfrac{\alpha'}{2}\dfrac{U}{m}\right)}, \tag{3}$$

where x_0 is the zero mode of X and the determinant is taken over the μ and ν indexes.

The expression (3) is divergent, using ζ-function regularization [11] one finds:

$$Z(g, F, T_0, U) \propto e^{-T_0} \sqrt{\det\left(\frac{g + 2\pi\alpha' F}{\pi\alpha' U}\right)} \times$$

$$\times \det\Gamma\left(1 + \frac{\alpha' U/2}{g + 2\pi\alpha' F}\right), \tag{4}$$

where $\Gamma(g)$ is the Γ-function. The dependence of the transcendental functions on the matrix U is assumed to be defined by their Taylor expansion. The divergence in (4) as $U \to 0$ is due to the infinite volume of the D-brane (and becomes a volume factor in that limit).

We would like to consider interactions of the D25-brane (1) with massless closed string fields. For example the D25-brane tension can be extracted from the expectation value of the graviton vertex operator. Consider the correlator:

$$\left\langle \int_D d^2\sigma h_{\mu\nu} \partial^a X^\mu(\sigma) \partial^a X^\nu(\sigma) \right\rangle_{F, T_0, U}, \tag{5}$$

where the averaging taken in the functional integral (1). $h_{\mu\nu}$ is a constant traceless matrix which defines the polarization of the graviton, and we could consider in exactly the same manner correlators corresponding to the anti-symmetric tensor field B or to the dilaton.

It is easy to see that (5) is given by:

$$\left\langle \int_D d^2\sigma h_{\mu\nu} \partial^a X^\mu(\sigma) \partial^a X^\nu(\sigma) \right\rangle_{F, T_0, U} =$$

$$= h_{\mu\nu} \left(\frac{\delta Z(g + h', F, T_0, U)}{\partial h'_{\mu\nu}}\right)_{h'=0} =$$

$$= -Z(g, F, T_0, U) \sum_{m=1}^{\infty} \mathrm{Tr}\left[\frac{h}{g + 2\pi\alpha' F + \frac{\alpha'}{2}\frac{U}{m}}\right], \tag{6}$$

where the trace is taken over the μ and ν indices.

3. We can compare this computation with a naive application of the boundary state formalism.

The boundary state $|B\rangle$ is a quantum state of closed string theory which obeys the boundary condition (2):

$$\Bigg(g_{\mu\nu}\, \partial_n \hat{X}^\nu(\theta) + 2\pi\alpha' F_{\mu\nu} \partial_\theta X^\nu(\theta) +$$

$$+ \frac{\alpha'}{2} U_{\mu\nu} \hat{X}^\nu(\theta)\Bigg) |B\rangle = 0, \tag{7}$$

where $\hat{X}(\theta)$ is the operator corresponding to the boundary value of the map X with the following closed string mode expansion:

$$\hat{X}^\mu(z,\bar{z}) = x_0^\mu + p^\mu \log z + \sum_{n\neq 0}\left[\frac{\alpha_n^\mu}{n} z^n + \frac{\tilde{\alpha}_n^\mu}{n}\bar{z}^n\right]. \quad (8)$$

In this formula $z = e^{-\sigma_1 - i\sigma_2} = \rho e^{-i\theta}$, $0 \le \rho \le 1$ is the complex coordinate on the disc, x_0 and p are coordinate and momentum of the string center of mass, the sum runs over n from minus infinity to plus infinity except zero and the generators α and $\tilde{\alpha}$ obey certain conditions to make $\hat{X}$ hermitian, as well they obey the standard commutation relations (see e.g., [24, 25]).

The solution to (7) is

$$|B\rangle = \mathcal{N}\times \times\prod_{n\ge 1}\exp\left\{-\left[\frac{g - 2\pi\alpha' F - \frac{\alpha'}{2}\frac{U}{n}}{g + 2\pi\alpha' F + \frac{\alpha'}{2}\frac{U}{n}}\right]_{\mu\nu}\frac{\alpha_{-n}^\mu\tilde{\alpha}_{-n}^\nu}{n}\right\}|0\rangle, \quad (9)$$

where $|0\rangle$ is the vacuum state, which is annihilated by all creation operators α_n and $\tilde{\alpha}_n$ with $n > 0$, $\mathcal{N}$ is a normalization constant.

The normalization is fixed by considering the coupling of the off-shell (momentum zero) closed string tachyon whose coupling to the boundary state should be equal to a trivial perturbation of the sigma model partition function,

$$\mathcal{N} = \langle 0|\, B\rangle = Z(g, F, T_0, U). \quad (10)$$

Now we would like to find (along the lines of [24, 25]) the reaction of the background closed string fields on the state $|B\rangle$. For this we consider the correlator:

$$\langle h|\, B\rangle = \langle 0|\alpha_1^\mu\tilde{\alpha}_1^\nu h_{\mu\nu}|B\rangle, \quad (11)$$

which should be compared with the correlator (5). However, we obtain

$$\langle h|B\rangle = = -Z(g,F,T_0,U)\left[\frac{g - 2\pi\alpha' F - \frac{\alpha'}{2}U}{g + 2\pi\alpha' F + \frac{\alpha'}{2}U}\right]^{\mu\nu} h_{\mu\nu} = = -2\, Z(g,F,T_0,U)\,\mathrm{Tr}\left[\frac{h}{g + 2\pi\alpha' F + \frac{\alpha'}{2}U}\right], \quad (12)$$

where at the last step we used the fact that h is traceless.

The formula (6) clearly does not agree with (12). Note that they would agree if $U = 0, \infty$ [24, 25]. In fact, these two expressions agree up to a (infinite) normalization factor at the fixed points of the renormalization group flow, $U \to 0$ and $U \to \infty$. As we will see below the infinite factor will turn out to be the volume of the non-compact group PSL(2,R) [27].

4. The apparent paradox that we have arrived at should not be surprising. The application of the boundary state formalism to the computation of the expectation value of a closed string vertex operator in open string theory requires a conformal mapping of the punctured disc, which is the world-sheet of open strings, to the semi-infinite cylinder, which is the world-sheet of closed strings. In the conformally non-invariant theory that we are considering here, it is natural to expect that this mapping is blocked by the conformal anomaly.

The global conformal group of the disc is PSL(2,R)[1]. If the PSL(2,R) symmetry were *not* broken it would be possible to use it to fix the position one point on the disc and one point on its boundary (or three points on the boundary). This could be used to get rid of integration over σ in (5). This means that:

$$\left\langle\int_D d^2\sigma h_{\mu\nu}\partial^a X^\mu(\sigma)\partial^a X^\nu(\sigma)\right\rangle_{F,T_0,U=0} = = \pi\,\langle h_{\mu\nu}\partial^a X^\mu(\sigma')\partial^a X^\nu(\sigma')\rangle_{F,T_0,U=0}. \quad (13)$$

Here σ' is some particular point on the disc, say 0. We expect that this will occur when $U = 0$ or $U = \infty$. However, since the conformal symmetry is broken when U does not have these values, the matrix element depends on the position and the integration is important.

The conformal mapping of a point z on the disc to a point η in the cylinder is $z = e^{-\eta}$. In this mapping, the center of the disc, at point $z = 0$ is mapped to the cap of the cylinder at infinity, $\mathrm{Re}\eta = \infty$. In the boundary state computation which leads to (11) it is assumed that the boundary state is at one cap of the cylinder, where $\mathrm{Re}\eta = 0$ and the graviton $|h\rangle$ is at the other cap which is located at $\mathrm{Re}\eta = \infty$, which is the image of the center of the disc. For this reason, we expect the boundary state computation to produce the expectation value of the graviton vertex operator inserted at the center of the disc.

In the sigma model, it is straightforward to compute the correlator:

$$\langle h_{\mu\nu}\partial^a X^\mu\,\partial^a X^\nu(\rho,\theta)\rangle_{F,T_0,U} \quad (14)$$

by summing the perturbation expansion for U, similar to computations in refs. [26, 28]. In the course of the

[1] The relevance of PSL(2,R) in a similar context was previously noticed in [4] and in [12].

calculation we use the boundary-to-disc propagator with Neuman boundary conditions:

$$G(\rho e^{i\theta}, e^{i\theta'}) = \frac{1}{\pi}\sum_{m=1}^{\infty}\frac{\rho^m}{m}\cos\left[m\left(\theta-\theta'\right)\right], \quad (15)$$

which also gives the boundary-to-boundary propagator in the limit $\rho \to 1$. Explicitly the contribution to the correlator from n interactions with the background U is:

$$\begin{aligned} &Z(g,F,T_0,U)\mathrm{Tr}\left(\frac{-\alpha' U}{2}\right)^n \times \\ &\times \frac{1}{\pi^n}\int d\phi_1 \dots d\phi_n \left(\partial_\rho\hat{\rho} + \frac{1}{\rho}\partial_\phi\hat{\phi}\right) \times \\ &\times \sum_{m=1}^{\infty}\frac{\rho^m \cos[m(\phi-\phi_1)]}{m} \times \\ &\times \sum_{m_1=1}^{\infty}\frac{\cos[m_1(\phi_1-\phi_2)]}{m_1}\dots\left(\partial_\rho\hat{\rho} + \frac{1}{\rho}\partial_\phi\hat{\phi}\right) \times \\ &\times \sum_{m_n=1}^{\infty}\frac{\rho^{m_n}\cos[m_n(\phi_n-\phi)]}{m_n}. \end{aligned} \quad (16)$$

The above integral is trivial, and upon summation over all values of n and inclusion of the antisymmetric field F the result is:

$$\begin{aligned} &\langle h_{\mu\nu}\partial^a X^\mu \partial^a X^\nu(\rho,\theta)\rangle_{F,T_0,U} = -2\,Z(g,F,T_0,U) \times \\ &\times \sum_{m=1}^{\infty} m\,\mathrm{Tr}\left(\frac{h}{g+2\pi\alpha' F + \frac{\alpha'}{2}\frac{U}{m}}\right)\rho^{2(m-1)}. \end{aligned} \quad (17)$$

Now it is easy to see that to obtain (6) one has to integrate this expression with the measure $\int \rho\, d\rho\, d\theta$, while to obtain (12) it is necessary to put $\rho = 0$: only the $m = 1$ term survives in this case. This is in agreement with our expectation that the boundary state describes the matrix element only when the operator is inserted at the center of the disc.

5. With the above choice of coordinates on the disc, there is a subset of the full conformal group of the plane which preserves the position and shape of the boundary of the disc. This subset is a PSL(2,R) subgroup of the full conformal group. It acts on the complex coordinates of the disc as

$$z \to w(z) = \frac{az+b}{b^* z + a^*}, \quad (18)$$

where

$$|a|^2 - |b|^2 = 1. \quad (19)$$

It is easy to verify that the unit circle is mapped onto itself. Thus, this mapping preserves the boundary of the disc. The origin is mapped to the point $b/a^* = \rho e^{-i\theta}$ in the interior of the disc.

We will examine how the boundary state $|B\rangle$ behaves under this transformation.

The boundary state is created by the exponential of the operator

$$\begin{aligned} &\mathcal{B} = \sum_{n=1}^{\infty} S_{\mu\nu}(n)\alpha^{\mu}_{-n}\tilde{\alpha}^{\nu}_{-n}, \quad \text{and} \\ &S_{\mu\nu}(n) = \frac{1}{n}\left[\frac{g-2\pi\alpha' F - U/n}{g+2\pi\alpha' F + U/n}\right]_{\mu\nu}, \end{aligned} \quad (20)$$

where α^μ_n and $\tilde{\alpha}^\mu_n$ are closed string oscillators. It is useful to write this operator in terms of position variables. For this, we introduce the two fields,

$$A^\mu(z) = \sum_{n\neq 0}\frac{\alpha^\mu_{-n}}{n} z^n, \quad (21)$$

$$\tilde{A}^\mu(\bar{z}) = \sum_{n\neq 0}\frac{\tilde{\alpha}^\mu_{-n}}{n}\bar{z}^n. \quad (22)$$

Here z are complex coordinates on the disc and $\eta = -\ln z$ are coordinates on a cylinder.

Then (20) can be written in the form

$$\mathcal{B} = \oint \frac{dz}{2\pi i}\oint \frac{d\bar{z}}{-2\pi i} S_{\mu\nu}(z,\bar{z}) A^\mu(z)\tilde{A}^\nu(\bar{z}), \quad (23)$$

where the integrations are on the unit circle and the kernel is defined by the power series

$$S_{\mu\nu}(z,\bar{z}) = \sum_{p=1}^{\infty} S_{\mu\nu}(p)\; p^2\; (z\bar{z})^{p-1}. \quad (24)$$

Now, we take into account that, under a general coordinate transformation, the coordinate functions $A^\mu(z)$ and $\tilde{A}(z)$ transform like,

$$A'^\mu(z') = A^\mu(z), \quad \tilde{A}'^\mu(\bar{z}') = \tilde{A}^\mu(\bar{z}). \quad (25)$$

If we apply this equation to the conformal transformation and change variables in the integral, we obtain the transformed boundary operator:

$$\mathcal{B}(a,b) = \oint \frac{dz}{2\pi i}\oint \frac{d\bar{z}}{-2\pi i} S_{\mu\nu}(z,\bar{z}|a,b) A^\mu(z)\tilde{A}^\nu(\bar{z}), \quad (26)$$

where

$$S_{\mu\nu}(z,\bar{z}|a,b) = \left|\frac{dw}{dz}\right|^2 S_{\mu\nu}(w(z),\bar{w}(\bar{z})). \quad (27)$$

As an exercise, we can verify that the usual, conformally invariant boundary state would be independent of the PSL(2,R) coordinates (a,b). In that case (for simplicity we put here $F=0$),

$$S^0_{\mu\nu}(p)=\delta_{\mu\nu}/p \tag{28}$$

and

$$S^0_{\mu\nu}(z,\bar z)=\delta_{\mu\nu}/(1-z\bar z)^2, \tag{29}$$

$$S^0_{\mu\nu}(z,\bar z|a,b)= = \delta_{\mu\nu}\left|\frac{dw(z)}{dz}\right|^2\frac{\delta_{\mu\nu}}{(1-w(z)\bar w(\bar z))^2}=\frac{\delta_{\mu\nu}}{(1-z\bar z)^2}.$$

This is independent of a and b, which is the desired result.

Then, the PSL(2,R) transformed boundary state is created by the exponential of the operator $\mathcal{B}(a,b)$. In terms of oscillators, this operator has the form

$$\mathcal{B}(a,b)=\sum_{m,n>0}S_{\mu\nu}(m,n|a,b)\alpha^\mu_{-m}\tilde\alpha^\nu_{-n}. \tag{30}$$

(We will verify that it still contains only negative index oscillators.) The moments are defined by

$$S_{\mu\nu}(m,n|a,b)= =\sum_{p=1}^{\infty}\frac{p}{m}\frac{p}{n}S_{\mu\nu}(p)\oint\frac{dz}{2\pi i}z^{p-1}\left(\frac{b^*z+a^*}{az+b}\right)^m\times \times\oint\frac{d\bar z}{-2\pi i}\bar z^{p-1}\left(\frac{b\bar z+a}{a^*\bar z+b^*}\right)^n. \tag{31}$$

Since $|a|/|b|>1$, the contour integrals on the right-hand-side of (31) have poles inside the unit circle only when $m,n>0$. Therefore they are non-zero only when $m>0$ and $n>0$, as anticipated in (30). It is straightforward to evaluate the integrals in (31). For example, in the case which we will see shortly is relevant to massless closed string states,

$$S_{\mu\nu}(1,1|a,b)=\frac{1}{|a|^2}\sum_{p=1}^{\infty}\left|\frac{b}{a}\right|^{2p-2}p^2\;S_{\mu\nu}(p)\;. \tag{32}$$

We can see from the form of the transformation in (31) that the boundary states generally depend on all three parameters of PSL(2,R). In some special cases, (32) for example, it depends on fewer parameters, such as $|b/a^*|=\rho$. The matrix element of any massless closed string state with the boundary state will depend on the PSL(2,R) parameters only through this dependence on the coordinate.

Note that the matrix element $\langle 0|\,B\rangle$ does not change under the transformation (18), i.e. the eq. (10) is legitimate. However, the correlator $\langle h|\,B\rangle$ transforms according to (32) as:

$$\langle h|\,B_\rho\rangle=\langle 0|h_{\mu\nu}\alpha^\mu_1\tilde\alpha^\nu_1\,Z(g,F,T_0,U)\times \times\exp\left\{-\sum_{m>0}\frac{1}{m}\left[\frac{g-2\pi\alpha' F-\dfrac{\alpha' U}{2m}}{g+2\pi\alpha' F+\dfrac{\alpha' U}{2m}}\right]_{\mu\nu}\times \times m^2(1-\rho^2)^2(-\rho)^{2(m-1)}\alpha^\mu_{-1}\tilde\alpha^\nu_{-1}+...\right\}|0\rangle= =-2Z(g,F,T_0,U)\times \times\sum_m m\mathrm{Tr}\left[\frac{h}{g+2\pi\alpha' F+\dfrac{\alpha' U}{2m}}\right]\rho^{2(m-1)}(1-\rho^2)^2. \tag{33}$$

It is worth mentioning here that if $U\to 0$ then

$$\sum_{m>0}m\rho^{2(m-1)}=\frac{1}{(1-\rho^2)^2}$$

exactly cancels $(1-\rho^2)^2$ in the numerator and, hence, (6) agrees with (33), (12).

At the same time the Haar measure on the PSL(2,R) group is given by:

$$\int d^2a d^2b\,\delta(|a|^2-|b|^2-1)f= =\int d^2a d^2b\,\delta(|a|^2-|b|^2-1)\times \times\int d\rho\,\delta\left(\left|\frac{b}{a}\right|-\rho\right)f=2\,\pi^2\int\frac{\rho\,d\rho}{(1-\rho^2)^2}f, \tag{34}$$

which is valid if the function f within the integral depends only on ρ. Combining the formulas (33) and (34) we find exact agreement.

In conclusion, we conjecture that, the average over PSL(2,R) of the transformed boundary state,

$$|\hat B\rangle=\int d^2a d^2b\,\delta(|a|^2-|b|^2-1)\,|B_{a,b}\rangle, \tag{35}$$

will have the correct overlap with any on-shell closed string state. Here we have checked this for the closed string tachyon, the graviton and it is straightforward to check it for the anti-symmetric tensor which has a non-zero expectation value when a background gauge field is turned on. It would be interesting to check this hypothesis for higher order correlation functions. The generalization of our results to the superstring boundary states with linear tachyon profile is straightforward.

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