Chiral theory of nucleons

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(Submitted 2 December 1985)
Pis'ma Zh. Eksp. Teor. Fiz. **43**, No. 2, 57–59 (25 January 1986)

An effective chiral Lagrangian containing all powers of the gradients of the pion field is constructed. A new formulation is offered for the problem of the nucleon as a bound state of quarks in a self-consistent field described by a chiral Lagrangian. Preliminary estimates yield satisfactory values for the mass and size of the nucleon.

Skyrme,[1] Witten,[2] and Adkins *et al.*[2] have suggested treating the nucleon as a topological soliton of a chiral Lagrangian (see the review in Ref. 3). This idea, however, could be pursued only in the Skyrme model, since the form of the complete chiral Lagrangian was not known. In the present letter we suggest a new approach to the nucleon, which is similar to the soliton approach but which has a clear physical meaning and which allows a parametrically justified calculation of the mass, size, and other characteristics of baryons.

To construct a chiral Lagrangian containing all powers of the gradients of the pion field, it is first necessary to have a theory for the spontaneous breaking of chiral invariance. It has been shown[4–6] that a sufficiently rarefied instanton vacuum of QCD provides a good explanation for this extremely important aspect of strong interactions. In particular, we derived[6] an effective low-energy action for Goldstone fields, $S[\pi]$. This action has the form of a functional determinant of the Dirac operator of a quark with a dynamic mass M in an external pion field:

$$S[\pi] = \ln \det [i\hat{\partial} + iM \exp (i\pi^a(x)\tau^a\gamma_5)] . \tag{1}$$

This expression holds at momenta $p < 1/\bar{\rho}$ of the pion field, where $\bar{\rho}$ is the average size of the instantons. At momenta $p \gtrsim 1/\bar{\rho}$, other hadronic degrees of freedom become important, and it is necessary to take into account the momentum dependence of the dynamic mass. We note that $M(p)$ falls off rapidly at[5,6] $p > 1/\bar{\rho}$, so that at high momenta we are dealing with a theory of free massless quarks (with perturbative corrections). The values of $M = M(p = 0)$ and $1/\bar{\rho}$ in an instanton vacuum are found from the QCD parameter Λ (in practice, they are determined from the gluon condensate $\langle F_{\mu\nu}^2 \rangle$) and are $\sim$345 MeV and $\sim$600 MeV, respectively.[5,6] We wish to stress that theoretically $M(0)$ is small, on the order of the packing parameter of the instanton medium. It is therefore precisely the rarefied instanton vacuum of QCD that makes it a meaningful problem to construct a nontrivial chiral Lagrangian which is applicable over a broad momentum range.[1)]

We consider a current correlation function of N_c quarks (N_c is the number of colors) which have the quantum numbers of a nucleon and which are separated by a large Euclidean time T. The asymptotic expression for the correlation function gives us the mass of the lowest state (the nucleon), $\mathcal{M}_N$:

$$\Pi_N(T) = \langle \underbrace{\psi(0,0)\dots\psi(0,0)}_{N_c};\ \underbrace{\psi^+(0,T)\dots\psi^+(0,T)}_{N_c}\rangle = \frac{1}{\mathcal{N}}\int D\pi \int D\psi D\psi^+$$

$$\times \underbrace{\psi\dots\psi(0)}_{N_c}\ \underbrace{\psi^+\dots\psi^+(T)}_{N_c} \exp\{\int d^4x\, \psi^+[i\hat{\partial} + iM\exp(i\pi\gamma_5)]\psi\} \underset{T\to\infty}{\to} \exp(-\mathcal{M}_N T). \tag{2}$$

In accordance with the discussion above, we retain from all of QCD only two degrees of freedom: the Goldstone field of pions and the quarks with a dynamic mass $M(0)$. We are assuming that the important momenta of the fields in integral (2) will turn out to be smaller than $1/\bar{\rho}$ [we recall that the size of the nucleon is $\sim(300\ \mathrm{MeV})^{-1}$]. The integral over ψ and ψ^+ in (2) is the product of the quark propagator in the pion field (raised to a power of N_c) and the determinant of the Dirac operator in (1). If the pion field is stationary, in the limit $T\to\infty$ the first is determined by the lower level with a positive energy of the Dirac Hamiltonian,

$$H = i\partial_i\gamma_0\gamma_i + M\gamma_0\exp(i\pi^a(\mathbf{x})\tau^a\gamma_5), \tag{3}$$

while the second is determined by the entire negative continuum of the same Hamiltonian. In the limit $T\to\infty$ we thus have

$$\Pi_N(T) = \int D\pi(\mathbf{x}) \exp\{-TN_c[E_{min,>0} + \sum_{E_n<0}(E_n - E_n^{(0)})]\}. \tag{4}$$

At large values of N_c this integral can be evaluated by the method of steepest descent in the stationary field $\pi(\mathbf{x})$. Quantum fluctuations of $\pi(\mathbf{x}, t)$ around the saddle point make corrections $1/N_c$ to the mass of the nucleon. On the leading order in N_c, the mass of the nucleon is

$$\mathcal{M}_N = \min_{\{\pi(\mathbf{x})\}} N_c(E_l[\pi] + E_{\text{field}}[\pi]), \tag{5}$$

where E_l is the energy of the lower discrete level of Hamiltonian (3), which has come from the upper continuum, and E_{field} is the total energy of the lower continuum (minus the energy of the free quark) which is, according to (1),the energy of the pion field. Both quantities are functionals of $\pi(\mathbf{x})$. To find the classical part of the mass of the nucleon, it is necessary to minimize (5) with respect to $\pi(\mathbf{x})$.

The nucleon is thus a bound state of N_c quarks in a pion field; the formation of this state requires the expenditure of an energy E_{field}, which is given by chiral Lagrangian (1).

In the trivial case in which the saddle field $\pi(\mathbf{x})$ is a null field, we have $E_{\text{field}} = 0$, $E_l = M$ (the bottom of the upper continuum), and $\mathcal{M}_N = N_c M$; i.e., the "nucleon" consists of N_c free quarks. Analysis of the nonrelativistic case of small and slowly varying $\pi(\mathbf{x})$ shows that this situation is unstable; from the standpoint of the energy of the system, a development $\pi(\mathbf{x}) \sim 1$ and a variation of this field over a distance $\sim 1/M$ would be preferred. To calculate E_{field}, it is thus not sufficient to use the first terms in the expansion of $S[\pi]$ in powers of the gradients. A more accurate value is given for E_{field} by the interpolation formula

$$E_{\text{field}} \approx \int \frac{d^3\mathbf{p}}{(2\pi)^3} \operatorname{Tr}[U^+(\mathbf{p})U(-\mathbf{p})]\mathbf{p}^2$$
$$\times \int \frac{d^3\mathbf{k}\,d\omega}{(2\pi)^4} \cdot \frac{M^2}{(M^2+\omega^2+\mathbf{k}^2)(M^2+\omega^2+(\mathbf{k}+\mathbf{p})^2)}, \tag{6}$$
$$U(\mathbf{p}) = \int d^3\mathbf{x} \exp[i\mathbf{p}\cdot\mathbf{x} + i\pi^a(\mathbf{x})\tau^a],$$

which transforms into the exact expression under the conditions (1) $p \ll M$, (2) $p \gg M$, (3) small values of π^a but arbitrary p. Using the simplest, spherically symmetrical, single-parameter ansatz

$$\pi^a(\mathbf{x}) = \frac{x^a}{|x|} P(|x|), \quad P(r) = 2 \arctan \operatorname{tg}(r_0/r)^2, \tag{7}$$

along with (6), we find a local minimum of expression (5) with $r_0 \approx 0.6$ fm, $3E_l \approx 370$ MeV, $3E_{\text{field}} \approx 730$ MeV, and $\mathscr{M}_N \approx 1100$ MeV ($N_c = 3$). It is clear that a minimization of the trial function $P(r)$ on the basis of a larger number of parameters would yield a smaller value for the mass of the nucleon. Nevertheless, even this estimate appears to be quite reasonable. In particular, from asymptotic expression (7) for large distances we find the axial constant of the nucleon to be $g_A \approx 1.1$ (the experimental value is 1.23; the Skyrme model predicts[2] 0.61).

The pion stems from the spontaneous breaking of chiral invariance and would exist in a theory without confinement. It is extremely probable that the same is true of the nucleon.

We are deeply indebted to P. V. Pobylitsa for many discussions.

[1)]Expression (1) was recently offered as a hypothesis in Refs. 7 and 8, but without derivation and without any discussion of the range of applicability of (1).

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Translated by Dave Parsons