Double stimulated Mandelstam-Brillouin scattering as a wave-front-inversion mechanism

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Under nonlinear steady-state conditions of double stimulated Mandelstam-Brillouin scattering in the field of a phase-modulated pump wave a scattered Stokes wave has an inverted wave front.

Stimulated Mandelstam-Brillouin scattering in condensed media is widely used for wave-front inversion.[1] As a linear medium only a plasma can be used for a wave-front inversion in an intense laser beam with an intensity higher than the breakdown threshold. However, a wave-front inversion due to stimulated Mandelstam-Brillouin scattering in a plasma has been observed only in a few experiments, since this type of scattering usually has a convective instability, and the plasma dimensions are not large enough to generate a large amplification of the scattered field which is required for a wave-front inversion. These two deficiencies can easily cause a double stimulated Mandelstam-Brillouin scattering[2,3] (DSMBS) in a field of two oppositely directed waves if the plasma has a reflecting surface. In this letter we find the conditions at which the scattered Stokes waves produced by DSMBS have a wave front which is inverted relative to the wave front of incident light and we show that because of the low threshold and absolute nature of the instability, DSMBS may be the basis of a new method of wave-front inversion in an intense laser beam in a plasma.

We assume that the s-polarized electromagnetic radiation with a wavelength λ_0 is incident at an angle θ_0 on a plasma which is inhomogeneous along the X axis. We also assume that the plasma has a surface ($x = l$) which specularly reflects the electromagnetic waves that strike it. If the attenuation of sound is sufficiently large (γ_S is the damping constant, and ω_S is the frequency), and if the nonuniformity a of the wave front of the light incident on the plasma is relatively large

$$\lambda_0/a \ll \gamma_S/\omega_S \ll 1,$$

the truncated equations for slowly varying amplitudes of the incident (E_{01}), reflected (E_{0-1}), and scattered Stokes waves ($E_{-1\sigma}$) ($\sigma = \pm 1$) and of the sound waves (ν) can be written[2]

$$(\mathbf{k}_{0\sigma}\nabla)E_{0\sigma} = -(\omega_{Le}^2/2c^2)\nu E_{-1\sigma}, \quad (\mathbf{k}_{-1\sigma}\nabla)E_{-1\sigma} = (\omega_{Le}^2/2c^2)\nu^* E_{0\sigma}, \tag{1}$$

$$\left(\frac{\partial}{\partial t} + \gamma_S\right)\nu = \frac{\omega_S}{32\pi n_c \kappa_B T}\sum_{\sigma=\pm 1} E_{0\sigma}E^*_{-1\sigma}, \tag{2}$$

where $\mathbf{k}_{0\sigma}$ and $\mathbf{k}_{-1\sigma} = -\mathbf{k}_{0-\sigma}$ are the wave vectors of the pump waves and the

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scattered Stokes waves. In Eq. (1) we have dropped the time derivatives of the field amplitudes, under the assumption $\gamma_S L/c \ll 1$, and we have also dropped the second derivatives in the coordinates perpendicular to the direction of propagation of the waves, under the assumption that the typical nonuniformity of the plasma density, L, is small in comparison with the Fresnel length

$$L \ll l_F = a^2/\lambda_0 . \tag{3}$$

If the incident electromagnetic wave near the plasma boundary ($x = 0$) is modulated only with respect to the phase $E_{01}(x = 0,y,t) = E_0 \exp[i\psi(y \cos\theta_0)]$, and if there is no incoming Stokes wave $E_{-11}(x = 0,y,t) = 0$, the steady-state solutions of Eqs. (1) and (2) will have the form

$$\begin{aligned} E_{0\sigma}(x, y) &= E_0 e_{0\sigma}(x) \exp[i\psi(y\cos\theta_0 - (l + \sigma(x - l))\sin\theta_0)], \\ E_{-1\sigma}(x, y) &= E_0 e_{-1\sigma}(x) \exp[-i\Omega t - iQy - i\psi(y\cos\theta_0 + (l + \sigma(x - l))\sin\theta_0)], \end{aligned} \tag{4}$$

where Ω is the tuning of the frequency of the Stokes wave away from resonance, $\omega_S = 2k_0 v_S \sin\theta_0$, and $Q \ll \omega_0/c$. The solution of the equations for the functions $e_{\mu\sigma}(x)$, which follow from (1), (2), and (4) jointly with the boundary conditions, determines the distribution of the wave field in the layer and the magnitude of the frequency shift $\Omega(Q)$. Since the field of a scattered wave at the boundary $x = 0$, according to (4), is

$$E_{-1-1}(x = 0, y, t) = e_{-1-1}(0)\, e^{-i\Omega t - iQy} E^*_{01}(x = 0, y), \tag{5}$$

the wave front of a back-scattered wave ($Q = 0$) outside the plasma (for $x < 0$) is inverted with respect to the front of an incident pump wave. For $Q \neq 0$ the scattered wave propagates at an angle $\delta\theta \approx Q_c/\omega_0 \cos\theta_0$ in the direction opposite to the pump wave. A deviation in the propagation direction by an angle $\delta\theta$ causes the wave front of the scattered wave to rotate by the same angle. According to (5), the wave front of a scattered wave is similar to the wave front of an incident wave, their ratio being $\cos(\theta_0 + \delta\theta)/\cos\theta_0$. The nonlinear steady states of DSMBS [Eq. (4)] thus correspond to the radiation of scattered waves with an inverted wave front, regardless of the scattering angle $\delta\theta$.

Analysis of Eqs. (1) and (2) for a plasma slab of thickness l, which is uniform relative to the excitation of the steady states (4) gives rise to the following dispersion equation which determines the DSMBS threshold and the frequency shift Ω as a function of the parameter $\Delta = Ql \,\mathrm{ctan}\, \theta_0$:

$$\exp(\rho_2 - \rho_1) = [\beta(1 + r^2) - \rho_1 - i\Delta]/[\beta(1 + r^2) - \rho_2 - i\Delta], \tag{6}$$

where

$$\beta = \kappa/(1 - i\Omega/\gamma_S), \quad \rho_{1,2} = \frac{1 - r^2}{2}\beta \pm \left[\left(\frac{1 - r^2}{2}\right)^2 \beta^2 - \Delta^2 - i\Delta\beta(1 + r^2)\right]^{1/2},$$

r is the amplitude factor of the reflection of waves from the trailing boundary, and

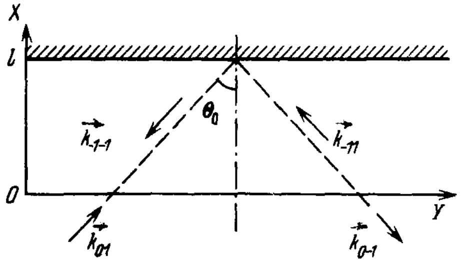

FIG. 1. The propagation of interacting waves in the case of DSMBS.

$\kappa = |E_0|^2 n_e \omega_S \omega_0^2\, l:(64\pi n_c^2 \kappa_B T k_{01x} \gamma_S)$ is the gain of a convective stimulated Mandelstam-Brillouin scattering.

Since the nonuniformity scale of the wave front of the incident light is not included in dispersion equation (6), the phase modulation in condition (3) has no effect on the DSMBS threshold. The minimum threshold, which is reached during back scattering ($Q = 0$), is equal to the minimum DSMBS threshold in the field of a pump wave with a plane wave front. The function $\kappa_{\rm th}(\Delta)$ for the case $r^2 = 1$ is shown in Fig. 2.

At $Q = 0$ the equations for the functions $e_{\mu\sigma}(x)$ are the same as the equations found in Ref. 2 for pump waves with a plane wave front. Even if the threshold is twice the normal threshold, the conversion of a phase-modulated pump wave into an inverted wave can therefore be expected to be $\approx r^2$.

Under the conditions of DSMBS there is an inversion of the wave front in a relatively thin layer [Eq. (3)] as a result of Bragg reflection of the incident light by sound which is excited by waves that propagate at an angle $2\theta_0$. To single out a first-order Bragg reflection corresponding to a wave with an inverted wave front, it is sufficient that the layer thickness satisfy the condition $L > \lambda_0/\theta_0^2$. In this respect, the inversion of a wave front as a result of DSMBS is similar to a wave-front inversion due to a four-wave mixing (see Chapter 6 in Ref. 1, for example), with the one exception that in our case, even a second "reference" wave with a conjugate wave front is produced during stimulated Mandelstam-Brillouin scattering. Under conditions of con-

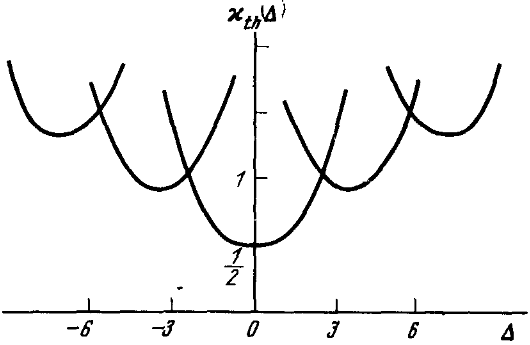

FIG. 2. The DSMBS threshold in a phase-modulated pump field versus the angle of propagation of scattered light in the case of a completely reflecting trailing boundary.

vective SMBS the three-dimensional structure, which provides the base for a wave-front inversion, is created by rays that propagate in nearly opposite direction to each other at small angles $\Delta\theta \sim a/\lambda_0$. To distinguish the components that are not inverted, the length of the nonlinear medium must be considerably greater, $L \gg a^2/\lambda_0$, which is difficult to achieve in a plasma.

If the incident beam is amplitude-modulated in the plasma, there may be wave-front inversion due to DSMBS. If, for example, the incident beam has "spots," whose size is $a \gg L$ and whose intensity is lower than the threshold intensity, then such regions will generally not be reproduced in the back-scattered beam. Furthermore, the nonlinear steady states (4) are unstable when the DSMBS threshold is higher than the normal threshold by a sufficient amount.[4] A wave-front inversion of good quality can therefore be achieved in a plasma when the DSMBS threshold is raised only moderately and when the amplitude modulation of the wave front of incident light is not too strong.

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