State density in gaps in the energy spectrum of 2*D* electrons in a transverse magnetic field

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The state density in the gaps of the energy spectrum between Landau levels (D_x) has been determined from an analysis of the thermally activated magnetoconductivity of the gas of 2*D* electrons in silicon metal-insulator-semiconductor structures and GaAs-$Al_x Ga_{1-x}$As structures. The value of (D_x) is shown to be determined by the product $\mu H \equiv \omega_c \tau$: $D_x = (2m/\pi\hbar^2)(1+\sqrt{\mu H})^{-1}$.

1. According to the present theoretical understanding, the energy spectrum of 2*D* electrons in a strong transverse magnetic field (H) consists of a set of Landau levels of width $\Gamma = \hbar\omega_c/\sqrt{(\pi/2)\mu H}$ ($\hbar\omega_c$ is the cyclotron energy, and μ is the mobility), while the state density in the gaps between levels is extremely low, determined by the tails of exponential functions, $D \sim \exp(-2E^2/\Gamma^2)$. In recent experiments carried out with GaAs-AlGaAs structures, involving an analysis of the heat capacity,[2,3] the magnetization,[4] and the thermally activated magnetoconductivity,[5,6] a common conclusion was reached: The state density in the gaps in the energy spectrum is approximately constant ($D = D_x$) and amounts to a rather significant fraction of the state density of 2*D* electrons under the conditions $H = 0$, $D_0 = g_v(m/\pi\hbar^2)$ (m is the mass of the state density, and g_v is the multiplicity of the valley degeneracy.

Our purpose in the present study was to determine the state density in the gaps of the energy spectrum, D_x, and its behavior as a function of μ and H for the system of 2*D* electrons in silicon metal-insulator-semiconductor structures, where the ability to vary the density (n) of 2*D* electrons provides the most direct method for determining the state density $D = dn/dE$.

2. Although the idea underlying the method of determining the state density through an analysis of the thermally activated magnetoconductivity is set forth in

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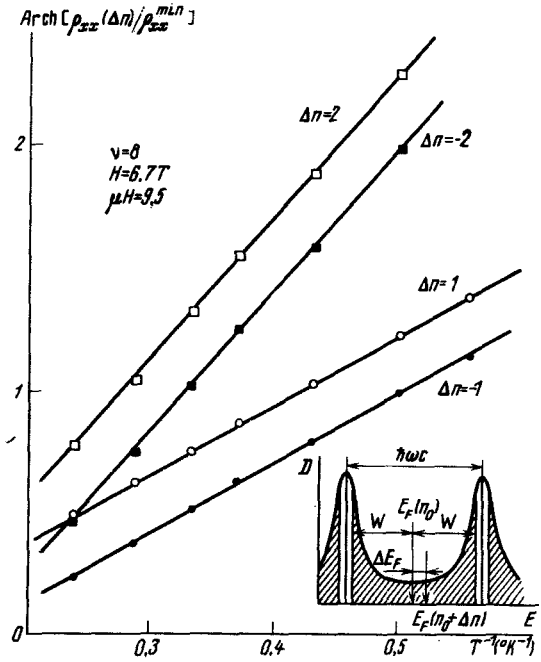

FIG. 1. Typical results on $\mathrm{Arch}(\rho_{xx}(\Delta n)/\rho_{xx}^{\min})$ as a function of the inverse temperature. $\nu = 8$, $H = 6.7$ T, $\mu(n_0) = 1.4\ \mathrm{m}^2/(\mathrm{V}\cdot\mathrm{s})$. The values shown for Δn in the figure are expressed in units of $10^{10}\ \mathrm{cm}^{-2}$. The inset is a sketch of the state density of $2D$ electrons in a strong transverse magnetic field. Shown here are the changes in the position of the Fermi energy E_F and the activation energies upon a change in the density, Δn; $\hbar\omega_c$ is the cyclotron energy. The hatching shows the region of localized states.

Refs. 5 and 6, it is necessary to simultaneously deal with the electron and hole components of the conductivity in order to find the correct value of D_x at the center of the gap during the data analysis, as we will show below. Since the electron states are localized in the gaps in the energy spectrum,[7] the magnetoconductivity σ_{xx} (and the magnetoresistance ρ_{xx}) is nonzero in this case (at temperatures that are not too low, $T > 1.5$ K), because of the thermal activation of electrons and holes from the Fermi level into bands of mobile states (see the inset in Fig. 1). When a given Landau sublevel becomes completely filled [i.e., at $n = n_0 = \nu(eH/2\pi\hbar)$, where e is the electron charge, and the integer ν is the filling factor], the magnetoconductivity reaches a minimum and is described by

$$\sigma_{xx}^{\min} \sim \rho_{xx}^{\min} \sim \exp(-W/kT), \tag{1}$$

where the activation energy W is equal to half the size of the energy gap. Upon a change Δn in the density from the value n_0, the Fermi level shifts a distance ΔE_F away from the middle of gap toward one of the Landau levels (see the inset in Fig. 1). In this case, when the Fermi energy is shifted ΔE_F from the middle of the gap, it is necessary to consider the electron component ($\sigma_{xx}^{e} \sim \exp[-(W-\Delta E_F)/kT]$ and

the hole component ($\sigma_{xx}^{h} \sim \exp[-(W+\Delta E_F)/kT]$ of the magnetoconductivity (especially under the condition $\Delta E_F \lesssim kT$). Consequently, we find, for the Landau ground level, for example,

$$\sigma_{xx}(\Delta n) \sim \exp(-W/kT)\operatorname{ch}(\Delta E_F/kT). \tag{2}$$

From (1) and (2) we find

$$\Delta E_F/kT = \operatorname{Arch}(\sigma_{xx}(\Delta n)/\sigma_{xx}^{\min}) \sim \operatorname{Arch}(\rho_{xx}(\Delta n)/\rho_{xx}^{\min}). \tag{3}$$

A study of the temperature-dependent magnetoconductivity can thus yield the dependence of ΔE_F on Δn, according to (3), and by differentiating we can find the state density in the gap of the energy spectrum of $2D$ electrons.

3. We studied six Si(100) metal-insulator-semiconductor structures, in which the peak mobility of the $2D$ electrons ranged from 3.6 to 1.3 $m^2/(V \cdot s)$, and also GaAs-$Al_{0.3}Ga_{0.7}As$ structures with $n = (3\text{–}5) \times 10^{11}$ cm^{-2} and $\mu = 5\text{–}8$ $m^2/(V \cdot s)$. The basic parameters of the structures have been published elsewhere.[8,9] The measurements were taken with an alternating current at a frequency of 20 Hz in a magnetic field $H \leqslant 20$ T.

Figure 1 shows some typical results on Arch $[\rho_{xx}(\Delta n)/\rho_{xx}^{\min}]$ as a function of the inverse temperature for various positive and negative values of Δn and for $\nu = 8$, $H = 6.7$ T, and $\mu = 1.4$ $m^2/(V \cdot s)$. We see that when plotted in these coordinates, the experimental points conform well to straight lines, whose slopes determine the change

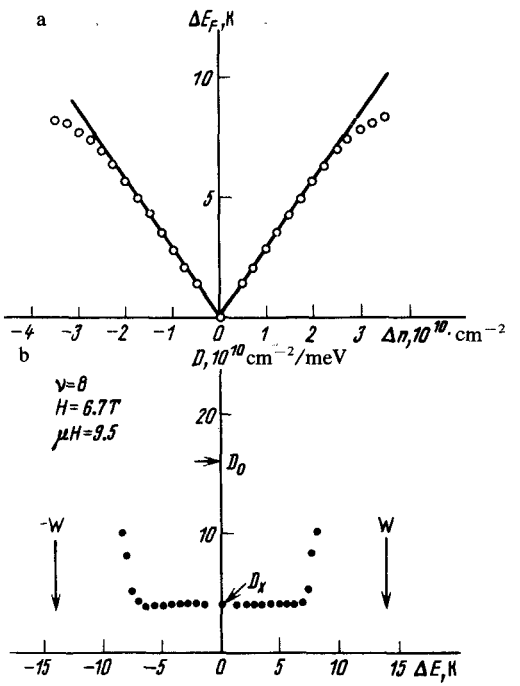

FIG. 2. a: ΔE_F as a function of the change in the density of $2D$ electrons, Δn. b: State density D as a function of the electron energy for $\nu = 8$, $H = 6.7$ T, and $\mu = 1.4$ $m^2/(V \cdot s)$. The arrows show the values of D_0 (the state density of $2D$ electrons at $H = 0$) and D_x (the residual state density of $2D$ electrons in a magnetic field).

in Fermi energy, ΔE_F, for the given Δn. The dependence of ΔE_F on Δn found in this manner is shown in Fig. 2a for positive and negative values of Δn. We see that at small values of Δn the experimental points lie on two symmetric rays coming from the origin. This situation corresponds to a constant state density $D = dn/dE_F$ near the middle of the gap in the energy spectrum of $2D$ electrons [in contrast with the result of Ref. 5, where an additional maximum in the state density was observed at the middle of the energy gap; that maximum stemmed from the disregard of one of the exponential functions in (2)]. As Δn is increased, the experimental curves of $\Delta E_F(\Delta n)$ bend downward, implying an increase in the state density as the Fermi energy approaches the middle of the Landau level.

Figure 2b shows a plot of the state density versus the energy in the gap in the energy spectrum between Landau levels found by differentiating the curves of $\Delta E_F(\Delta n)$ for $\nu = 8$ and $H = 6.7$ T. We see that there is a rather broad interval of energies in which we have $D = D_x = \text{const}$, and D_x is a significant fraction of D_0. Similar curves of $D(E)$ were found for all six metal-insulator-semiconductor structures at the various values of $H(\mu H > 4)$ for $\nu = 4$ and 8.

4. The question of most fundamental importance is the dependence of D_x on the magnetic field and on the mobility of the $2D$ electrons. It was concluded in Refs. 5 and 6 that D_x depends on μ but not on H. According to our data, at $H = \text{const}$ we have $D_x \sim \mu^{-1/2}$, while at $\mu = \text{const}$ we have $D_x \sim H^{-1/2}$, so that D_x depends on only the product μH. This dependence is shown in dimensionless coordinates in Fig. 3 in log-log scale. Shown along with data for metal-insulator-semiconductor structures in this figure are points that we found for $2D$ electrons in GaAs-$Al_{0.3}Ga_{0.7}As$ structures. These points agree well with the data on the metal-insulator-semiconductor structures, so that we have also reproduced all the experimental points from Refs. 5 and 6 in Fig. 3. The results found for the silicon metal-insulator-semiconductor structures and those for the GaAs-AlGaAs structures are seen to agree fairly well, and the typical slope of the plot of D_0/D_x versus μH in logarithmic coordinates is approximately 1/2; i.e., we

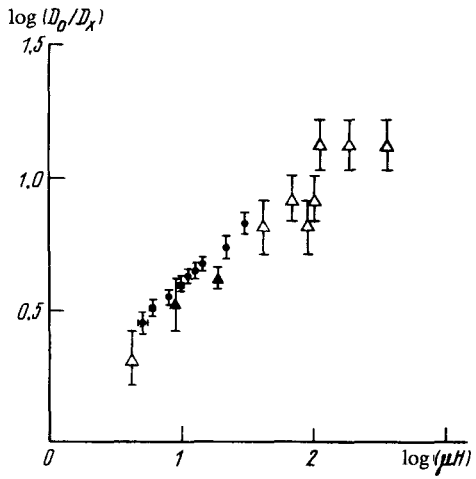

FIG. 3. The ratio of D_0 to D_x as a function of the product μH in log-log scale. Circles—metal-insulator-semiconductor structures ($\nu = 4$); squares—the same, but for $\nu = 8$; triangles—GaAs-$Al_xGa_{1-x}As$ structures; open symbols—experimental points from Refs. 5 and 6.

have $D_x/D_0 \sim (\mu H)^{-1/2}$. Analysis of our results for metal-insulator-semiconductor structures in the coordinates D_0/D_x, $(\mu H)^{1/2}$ reveals $D_x \simeq D_0(1+\sqrt{\mu H})^{-1}$.

5. In summary, on the basis of our analysis of the thermally activated magnetoconductivity of 2*D* electrons we have shown that (a) the state density is constant in the gaps in the energy spectrum of 2*D* electrons ($D = D_x = \text{const}$ at $\mu H > 4$), (b) D_x depends on both μ and H, and (c) the dependence of D_x on μ and H can be approximated well by the expression $D_x = D_0(1+\sqrt{\mu H})^{-1}$. This is a universal dependence that describes the results obtained from both silicon metal-insulator-semiconductor structures and GaAs-AlGaAs structures.

We wish to stress that the residual state density of 2*D* electrons found by us in a transverse magnetic field (D_x) depends on only the parameter $\mu H \equiv \omega\tau$. This result is in agreement with the results of Ref. 9, according to which the parameter μH also determines the fraction of localized states at the Landau level. We believe that the reason for the significant discrepancy between the theoretical and experimental values of D_x is that this quantity has been determined rigorously in the theory only for a weak, short-range scattering potential, while the actual potential is of a different nature.

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