Electrostatic oscillations in 2D systems with a quantum Hall effect

V. I. Tal'yanskiĭ
Institute of Solid State Physics, Academy of Sciences of the USSR

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In 2D systems of bounded dimensions (a 2D layer of circular shape or an ellipsoid cut out of a superlattice), under conditions corresponding to the quantum Hall effect, there are natural electrostatic oscillations which are conceptually analogous to the uniform precession of the magnetization in a ferromagnetic ellipsoid, as discussed by Kittel [Phys. Rev. **73**, 155 (1948)].

We consider a so-called bulk superlattice (e.g., one made from GaAs-AlGaAs; Ref. 2) consisting of alternating layers of different semiconductors. Two-dimensional electron channels (2D layers) arise at the interfaces between layers. In a magnetic field directed perpendicular to the 2D layers, the electrical conductivity of an individual 2D layer is described by the tensor

$$\hat{\bar{\sigma}} = \begin{pmatrix} \bar{\sigma}_{xx} & -\bar{\sigma}_{xy} & 0 \\ +\bar{\sigma}_{xy} & \bar{\sigma}_{xx} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

(the x and y axes are in the plane of the layer; the z axis runs perpendicular to the layer). Under conditions corresponding to the quantum Hall effect (the only case we will consider here) we have

$$\bar{\sigma}_{xx} \approx 0, \quad \bar{\sigma}_{xy} = e^2 n/2\pi\hbar, \tag{1}$$

where n is an integer or a fraction.

At sufficiently low frequencies we can ignore the frequency dependence of the quantities[3] $\bar{\sigma}_{xy}$; furthermore, if the scale lengths of the spatial variations in the fields and currents of the problem are much larger than the distance (d) between the 2D layers in the superlattice, the conductivity tensor for the entire superlattice is

$$\hat{\sigma} = \frac{1}{d}\hat{\bar{\sigma}}. \tag{2}$$

The electrodynamic properties of the superlattice can now be described by the dielectric tensor

$$\hat{\epsilon} = \begin{pmatrix} \epsilon_0 - \dfrac{4\pi i}{\omega}\sigma_{xx}; & \dfrac{4\pi i}{\omega}\sigma_{xy}; & 0 \\ -\dfrac{4\pi i}{\omega}\sigma_{xy}; & \epsilon_0 - \dfrac{4\pi i}{\omega}\sigma_{xx}; & 0 \\ 0 & 0 & \epsilon_0 \end{pmatrix}, \tag{3}$$

where ϵ_0 is the average dielectric constant of the semiconductors in the superlattice.

We cut an ellipsoid out of a superlattice. For an ellipsoidal sample we can relate the electric fields inside and outside the ellipsoid with the help of a depolarizing-factor tensor[4] (in the electrostatic approximation, curl $\mathbf{E} = 0$). The rest of the arguments are completely analogous for ellipsoids of any shape, so we will discuss here only the case of an ellipsoid of revolution with the axes (a,a,Δ). We assume that the $2D$ layers in the ellipsoid are perpendicular to the axis Δ. We assume that a static magnetic field [which leads to the satisfaction of conditions (1)] is directed along the z axis, which coincides with the ellipsoid axis Δ, and an external alternating electric field is directed along the x axis (x and y are parallel to the $2D$ layers):

$$\mathbf{E}_0 = \mathbf{x}_0 E_0 e^{i\omega t} . \tag{4}$$

The electric field inside the ellipsoid is then

$$\mathbf{E}^{(i)} = \mathbf{E}_0 - \hat{N}\mathbf{P}, \tag{5}$$

where the tensor of depolarizing factors is

$$\hat{N} = \begin{pmatrix} N_{\parallel} & 0 & 0 \\ 0 & N_{\parallel} & 0 \\ 0 & 0 & N_{\perp} \end{pmatrix}, \tag{6}$$

and the polarization is

$$\mathbf{P} = \frac{\hat{\epsilon} - 1}{4\pi} \mathbf{E}^{(i)} . \tag{7}$$

Substituting (7) into (5), and using (3), we find the equations

$$E_x^{(i)} \left(1 + N_{\parallel} \frac{\epsilon_0 - 1}{4\pi}\right) + N_{\parallel} \frac{i\sigma_{xy}}{\omega} E_y^{(i)} = E_{0x} , \tag{8}$$

$$- \frac{i\sigma_{xy} N_{\parallel}}{\omega} E_x^{(i)} + \left(1 + N_{\parallel} \frac{\epsilon_0 - 1}{4\pi}\right) E_y^{(i)} = 0 .$$

It follows from system (8) that free electrostatic oscillations ("free" here means even in the case $E_0 = 0$) should exist in the ellipsoid. We find the eigenfrequency ω_0 by equating the determinant of the system to zero:

$$\omega_0 = \sigma_{xy} \frac{N_{\parallel}}{1 + N_{\parallel} \frac{\epsilon_0 - 1}{4\pi}} . \tag{9}$$

Let us consider the case $\Delta \ll a$. In this case, the expression we have derived also holds for a thin disk (a and Δ are the diameter and height, respectively, of the disk) made up of $2D$ layers. Under the condition[4,5] $\Delta \ll a$ we have

$$N_{\|} \simeq 9\frac{\Delta}{a} \tag{10}$$

and $\omega_0 \simeq \sigma_{xy} N_{\|} = 9\sigma_{xy}(\Delta/a)$. If there are p two-dimensional layers in the disk, we have $\sigma_{xy} = \sigma_{xy}(p/\Delta)$ and

$$\omega_0 = 9\frac{p}{a}\sigma_{xy}. \tag{11}$$

The height of the disk, Δ, does not appear in (11), so there is the hope that (11) will also be a good approximation for the frequency of the electrostatic oscillations of an individual $2D$ layer in the form of a circle of diameter a:

$$\omega_0 \approx 9\frac{\sigma_{xy}}{a}.$$

For $\bar{\sigma}_{xy} \sim (10^4\ \Omega)^{-1}$ and $a \sim 1$ cm we have $\omega_0 \sim 10^8$–10^9 s^{-1}.

In this oscillation, the electric field is constant inside the ellipsoid, and the current is concentrated in the $2D$ layers (and is the same for any point belonging to the $2D$ layers) and directed perpendicular to the electric field. The uniform distribution of the field and the current rotates at an angular frequency ω_0. It is also easy to see the reason for the onset of the oscillations. The electric field in the sample causes a current to flow in the direction perpendicular to the field. The current causes charges to accumulate at the boundaries of the sample, and the field of these charges "rotates" the original electric field. The "rotated" field "rotates" the current; etc.

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