Low-frequency oscillations of a precessing magnetic domain in ^{3}He-*B*

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Two low-frequency oscillation modes of a precessing two-domain structure in ^{3}He-*B* are found. One of the modes, the bulk mode, is identified with the mode observed experimentally by Bun'kov *et al.* {Pis'ma Zh. Eksp. Teor. Fiz. **43**, 131 (1986) [JETP Lett. **43**, 168 (1986)]}. The other mode, the surface mode, is similar to the gravitational waves at the surface of a liquid.

A precessing two-domain structure in a superfluid ^{3}He-*B* (Ref. 1–3) is a minimum of a certain functional F which was determined previously.[3] If this structure is slightly perturbed, it will oscillate near the minimum. In the first experimental studies of a long-lived induction signal, carried out by Borovik-Romanov *et al.*,[1] a low-fre-

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quency modulation of this signal, which can be linked with these oscillations, was observed. More detailed studies of the oscillations of a two-domain structure that followed[4] have made it possible to identify these oscillations with one of the oscillation modes of a two-domain structure. In this letter we will identify low-frequency oscillations of such a structure that are theoretically feasible.

In the experiments of Borovik-Romanov *et al.*[1] and Bun'kov *et al.*,[4] both the magnetic field and its gradient are directed along the z axis which coincides with the axis of the cylindrical measuring chamber, while the domain wall is perpendicular to this axis. The two-domain structure that forms in the chamber is similar to the two-phase system in an external field directed along the z axis. A system of this sort can have bulk and surface oscillation modes. The states corresponding to a regular spin precession are degenerate in the precession phase—degenerate in the angle α. This degeneracy accounts for the presence of a gap-free bulk oscillation mode. The dispersion law for this mode can be determined by means of a standard linearization of the equations of motion with respect to the small increments ψ, η, and φ to the variables α, β, and Φ which describe the motion of the order parameter (the reader is referred to Ref. 3 for more details). The experimental data of Ref. 4 can be interpreted by analyzing just the oscillations with the wave vector $\mathbf{k}\|\hat{z}$. In this case we obtain the following equation for the frequency[1)]:

$$\omega = \left[\frac{2\Omega^2}{8\Omega^2 + 3\omega_L^2}(5c_\perp^2 - c_\parallel^2)\right]^{1/2} k. \qquad (1)$$

Here ω_L is the Larmor frequency, Ω is the longitudinal oscillation frequency, and $c_\parallel$ and $c_\perp$ are the coefficients in the gradient energy which represent the spin-wave velocities. This frequency must correspond to one of the natural oscillations of the domain. These oscillations can be determined if the boundary conditions are known. The boundary condition at the chamber walls consists of the absence of the magnetic-flux component normal to the wall—of the combination of the projections of the magnetization P. For the perturbations which depend only on z, this condition at the side chamber walls is satisfied automatically, but at the chamber bottom ($z = 0$) the following condition holds:

$$(5c_\perp^2 - c_\parallel^2)\,\psi' + 2(c_\parallel^2 - 2c_\perp^2)\varphi' = 0. \qquad (2)$$

The wavelength of the observable oscillations is comparable to the longitudinal dimension of the domain and is longer than the thickness λ of the domain wall. In the first approximation in $k\lambda \ll 1$, the wall may be assumed infinitely thin and the perturbations at the wall can be made to satisfy the boundary condition. To find this boundary condition, we must integrate over a small interval (z_1, z_2) which includes the domain wall and the law of conservation of P:

$$\frac{\partial}{\partial t}\int_{z_1}^{z_2} P dz = (1 - \cos\beta)\{2[c_\perp^2 + (c_\parallel^2 - c_\perp^2)\cos\beta]\alpha' - (2c_\perp^2 - c_\parallel^2)\Phi'\}\Big|_{z_1}^{z_2}. \qquad (3)$$

Defining the wall coordinates z_0 by $\int_{z_2}^{z_2} Pdz = 5\omega_L(z_2 - z_0)/4$ and letting z_1 and z_2 tend to z_0, we find

$$\frac{dz_0}{dt} = \frac{1}{2\omega_L}[(5c_\perp^2 - c_\parallel^2)\psi' + 2(c_\parallel^2 - 2c_\perp^2)\varphi']_{z_0-0}. \quad (4)$$

To eliminate the wall coordinates z_0 from this equation, we will make use of one more condition

$$\dot{\alpha}(z_0) = -\omega_L(z_0), \quad (5)$$

where $\omega_L(z_0)$ is the local Larmor frequency. This is a physically natural condition which can also be eliminated from the equation, in a manner similar to the elimination of the momentum conservation law, by following the procedure used in deriving (4). Condition (5), which corresponds to the instantaneous position of the wall, should be "transposed" in such a way that it would correspond to its mean position. Using Eq. (4), we thus find the boundary condition for $z = \bar{z}_0 = L$.

$$\ddot{\psi} = -\frac{\nabla\omega_L}{2\omega_L}[(5c_\perp^2 - c_\parallel^2)\psi' + 2(c_\parallel^2 - 2c_\perp^2)\varphi']. \quad (6)$$

A further analysis of the equations of motion shows that oscillations such as (1) change primarily the angle α; i.e., these oscillations correspond to a periodic "twisting" of the precessing spin into a spiral whose axis runs in the same direction as z. The increments η and φ are small in comparison with ψ to the extent that ω/Ω and ω/ω_L are small. Omitting therefore φ' in conditions (2) and (6), we find $\psi'(0) = 0$ and $\psi(L) = 0$. The solution $\psi \sim \cos(\omega t + \delta)\cos kz$ satisfies these conditions for $kL = \pi(n + 1/2)$, $n = 0,1,2,\ldots$. For the dominant mode we have $k = \pi/2L$; i.e., one-fourth of the wavelength corresponds to the length of the domain. This circumstance, along with dispersion law (1), means that the frequency of such oscillations is inversely proportional to L, consistent with the behavior observed experimentally by Bun'kov *et al.*[4] The frequency of the induction signal is given by $\dot{\alpha} = -\omega_L(L) + \dot{\psi}$, so that the oscillations also affect the modulation of the frequency of the induction signal. A quantitative comparison of the measured oscillation frequency with that calculated from Eq. (1), carried out in Ref. 4, shows that we see a dominant mode of the bulk oscillations of the precessing domain. The harmonics of the fundamental frequency and the oscillations with a radial component ψ have much higher frequencies than the fundamental frequency because of the geometry of the chamber used in the experiments of Bun'kov *et al.*,[4] so that these frequencies cannot be mistaken for the fundamental frequency.

Of all the surface modes concentrated near the domain wall, we consider only the mode with the lowest frequency. The principal variable quantity here is, as before, the angle α. The spatial dependence of the variable increment in this angle is

$$\psi \sim \exp\{iky + \kappa(z - z_0)\}.$$

This dependence satisfies the equations of motion when

$$\kappa = [(3c_\perp^2 + 5c_\parallel^2)/2(5c_\perp^2 - c_\parallel^2)]^{1/2}k.$$

Substituting the given solution into boundary condition (6), we obtain the dispersion law for the surface oscillations under consideration

$$\omega^2 = \frac{\nabla\omega_L}{2\sqrt{2}\omega_L}[(5c_\parallel^2 + 3c_\perp^2)(5c_\perp^2 - c_\parallel^2)]^{1/2}k. \qquad (7)$$

In other words, these oscillations are similar to the gravitational waves at the surface of a liquid. The properties of surface mode (7) are different from those of the oscillations studied experimentally by Bun'kov *et al.*[4] Detection and experimental study of this mode are tasks worth pursuing.

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[1)]In the limit $\Omega \ll \omega_L$, Eq. (1) gives the same result as the expression obtained elsewhere[5] for the perturbation frequency of a spatially homogeneous precession with $\beta \to \arccos(-1/4) + 0$.

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