

Friedel oscillations and Ruderman–Kittel interaction in disordered conductors

A. Yu. Zyuzin and B. Z. Spivak

A. F. Ioffe Physicotechnical Institute, Academy of Sciences of the USSR, Leningrad

(Submitted 26 December 1985)

Pis'ma Zh. Eksp. Teor. Fiz. **43**, No. 4, 185–187 (25 February 1986)

The temperature of the spin-glass phase transition cannot be found from the de Gennes–Mattis formula [P. G. de Gennes, *J. Phys. Radium* **23**, 630 (1962); D. C. Mattis, *Theory of Magnetism*, New York, 1965] for the Ruderman–Kittel interaction of localized spins in a disordered metal. The temperature of the spin-glass phase transition depends weakly on the concentration of nonmagnetic impurities. The spin-orbit scattering by nonmagnetic impurities gives rise to a non-Heisenberg interaction of the Dzyaloshinskii–Mori type between localized spins.

1. The existence of a Fermi surface in a metal or a heavily doped semiconductor is known to cause the magnitude of the exchange interaction of two spins, $I(\mathbf{r})$, separated by a distance r , to oscillate in accordance with¹

$$I(\mathbf{r}) = J^2 K_{zz}^0(\mathbf{r}, 0) = J^2 \frac{\nu_0}{4\pi} \frac{\cos 2p_F r}{r^3} \quad (1)$$

Here J is the constant of the interaction of the impurity spin with the conduction electrons, ν_0 is the state density at the Fermi level, p_F is the Fermi momentum, $K_{im}(\mathbf{r}, \mathbf{r}')$ is the spin susceptibility of the metal, given by

$$K_{im}(\mathbf{r}, \mathbf{r}') = T \sum_{\omega} \text{Sp} \{ \hat{\sigma}^i \hat{G}(\mathbf{r}, \mathbf{r}', \omega) \hat{\sigma}^m \hat{G}(\mathbf{r}', \mathbf{r}, \omega) \} \quad (2)$$

$\sigma_{\alpha\beta}^i$ are the Pauli matrices, $\omega = \pi T(2n + 1)$, $G_{\alpha\beta}(\mathbf{r}, \mathbf{r}', \omega)$ is the electron Green's function, α and β are spin indices, and T is the temperature.

The oscillations in (1) are a direct consequence of the Friedel oscillations of the electron wave functions which arise when a localized spin is inserted into a system. Another consequence of the Friedel oscillations is the fact that the electric field of the impurity in a metal is screened at large r in accordance with

$$\varphi_0(\mathbf{r}) \sim \frac{\cos 2p_F r}{r^3} \quad (3)$$

de Gennes² and Mattis³ have shown that in a slightly disordered metal expressions (2) and (3) become modified at $r > l$ (l is the electron mean free path in the case of scattering by nonmagnetic impurities, $p_F l \gg \hbar$):

$$\overline{K_{zz}(\mathbf{r})} = K_{zz}^0 l^{-r/l}; \quad \overline{\varphi(\mathbf{r})} = \varphi_0(r) l^{-r/l} \quad (4)$$

The Friedel oscillations are therefore damped exponentially over the mean free path l . The superior bar in (4) means an average over random realizations of the impurity potential. We will be making use of (4) in several places below to calculate the transition temperature T_c in spin glasses⁴:

$$T_c \approx J^2 \frac{\nu_0}{4\pi} \frac{1}{r_s^3} l^{-1} e^{-\frac{r_s}{l}} \quad (4a)$$

(r_s is the average distance between paramagnetic impurities). We will also be using (4) in calculations on the structures of amorphous and liquid metals.⁵ Experiments on the l dependence of T_c (Ref. 6) are usually also discussed with reference to (4) and (4a).

We can show that it is incorrect to use (4) and (4a) for these purposes. A qualitative explanation of this fact can be seen in the example of the Ruderman-Kittel interaction. The insertion of one paramagnetic center at the point $r = 0$ in a pure metal leads to oscillations of the electron wave functions and thus oscillations of the magnetization at the point \mathbf{r} : $M_0(r) \sim \cos 2 p_F r / r^3$. In a disordered metal, this expression becomes modified: $M(\mathbf{r}) \sim A(\mathbf{r}) r^{-3} \cos[2 p_F r + \delta(\mathbf{r})]$, where $A(\mathbf{r})$ is a smooth but otherwise random function, and $\delta(\mathbf{r})$ is the phase shift which is associated with the scattering of an electron by impurities. This shift becomes nearly random at $r > l$. When we take an average over the realizations of the random potential at $r > l$, i.e., over the random phase shifts $\delta(\mathbf{r})$, we find (4).

It is clear, however, that all of the phenomena listed above are determined by the typical values of $I(\mathbf{r})$, regardless of its sign. In a spin glass, e.g., T_c is determined by the quantity $\overline{(K_{im}^2(\mathbf{r}))^{1/2}}$. It is also obvious that we have $\overline{(K^2(\mathbf{r}))^{1/2}} \sim \overline{(M^2(\mathbf{r}))^{1/2}} \sim r^{-3}$ and that this quantity does not contain an exponentially small parameter. The estimate $T_c \approx J^2 \nu_0 / r_s^3$ in spin glasses which we find on the basis of these arguments differs by a factor $\exp(r_s/l) \gg 1$ from the standard estimate in (4a) (Refs. 4 and 6).

2. To find $K_{im}^2(\mathbf{r})$, we need to sum the sequence of Feynman diagrams shown in Fig. 1. The solid lines here correspond to electron propagators, while the dashed lines correspond to scattering by impurities. Similar diagrams are summed in a study of mesoscopic fluctuations in small samples.^{7,8}

Here are the results for the three-dimensional case at $T = 0$:

$$\overline{K_{im}^2(\mathbf{r})} = 32\pi\nu_0^2 \left\{ \Gamma(r, \tau_{s0}^{-1}) \left(\delta_{im} - \frac{1}{4} \right) + \frac{1}{4} \Gamma(r, 0) \right\}, \quad (5)$$

$$\Gamma(r, 0) = \frac{3}{2(4\pi)^3 r^6}; \quad \Gamma(r, \tau_{s0}^{-1}) = \begin{cases} \Gamma(r, 0) & r \ll L_{s0} \\ r^{-6} l - \frac{r}{L_{s0}} & r \gg L_{s0} \end{cases} \quad (6)$$

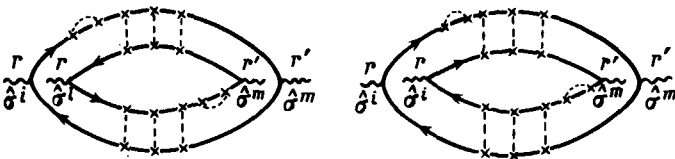


FIG. 1.

Here $L_{so} = \sqrt{D\tau_{so}}$, D is the electron diffusion coefficient, τ_{so} is the scale time for the spin-orbit scattering of electrons by nonmagnetic impurities. At $r \gg L_T = \sqrt{D/T}$ we have $\overline{K_{im}^2(\mathbf{r})} \sim \nu_0^2 r^{-6} \exp(-2r/L_T)$. In the limiting cases $l \ll r \ll L_{so} \ll L_T$ and $L_T \gg r \gg L_{so}$, the quantity $\overline{K_{im}^2(\mathbf{r})} \sim r^{-6}$ has a universal form, independent of l . Furthermore, at $r \gg L_{so}$, the quantity $\overline{K_{im}^2(\mathbf{r})}$ does not depend on the spin indices i and m . This result means that at $r > L_{so}$ the exchange between localized spins becomes anisotropic and is described by a random non-Heisenberg matrix of the Dzyaloshinskii-Mori type. This fact has already been used in explaining the l dependence of the hysteresis in the magnetization of spin glasses. The calculations which have been carried out, however, have been restricted to first order in $r/l < 1$.

The effect of an interaction between electrons on the value of $\overline{K_{im}(\mathbf{r})}$ was studied in Ref. 10. The results found there are higher than those found from (4) but parametrically similar than those found from (5).

It follows from (1) and (5) that in the intermediate region, $r \sim 1$, the coefficient of r^{-6} in the expression for $\overline{K^2(\mathbf{r})}$ changes by a factor of three, indicating an increase in the fluctuations of $A(\mathbf{r})$. We do not rule out the possibility that a 50% decrease in T_c in CuMn with decreasing l (Ref. 6) is a consequence of precisely this circumstance. The interpretation in Ref. 6 on the basis of (4a) leads to values 4.5 times smaller than those that follow from the conductivity.

Calculations analogous to those in (5) lead to

$$\overline{(\varphi^2(\mathbf{r}))}^{1/2} \sim r^{-3} \quad \text{at} \quad r \gg l. \quad (7)$$

Expressions (5) and (7) do not depend on l at large r , and they are valid in order of magnitude up to $p_F l \sim \hbar$. All of the effects described here stem from the circumstance that $K_{im}(\mathbf{r})$ is not self-averaging at $r > 1$. A situation analogous to that described above arises in a study of the fluctuations of the conductivity $\sigma(\mathbf{r}, \mathbf{r}')$ where we have

$$\overline{(\sigma^2(\mathbf{r}, \mathbf{r}'))}^{1/2} = \frac{3}{2^{3/2}\pi} \frac{e^2 D \nu_0}{|\mathbf{r} - \mathbf{r}'| l^2}, \quad (8)$$

while we also have¹¹

$$\overline{\sigma(\mathbf{r}, \mathbf{r}')} = \frac{\nu_0 e^2 D}{|\mathbf{r} - \mathbf{r}'|^2 l} - \frac{|\mathbf{r} - \mathbf{r}'|}{l}.$$

The latter circumstance must be taken into account in calculating the superfluid current in superconducting alloys.¹¹

We wish to thank B. L. Al'tshuler, A. G. Aronov, A. V. Gol'tsev, I. Ya. Korenblit, B. I. Shklovskii, and E. I. Shender for useful discussions.

¹M. A. Ruderman and C. Kittel, Phys. Rev. **96**, 99 (1954).

²P. G. De Gennes, J. Phys. Radium **23**, 630 (1962).

³D. C. Mattis, Theory of Magnetism, New York, 1965.

⁴A. A. Abrikosov, Adv. Phys. **29**, 869 (1980).

⁵A. Pasturel and J. Hartner, Phys. Rev. B **32**, 5009 (1985).

- ⁶M. Hitzfeld and P. Ziemann, *Phys. Rev. B* **32**, 3026 (1985).
- ⁷B. L. Al'tshuler, *Pis'ma Zh. Eksp. Teor. Fiz.* **42**, 291 (1985) [*JETP Lett.* **42**, 359 (1985)].
- ⁸P. A. Lee and D. Stone, *Phys. Rev. Lett.* **55**, 1622 (1985).
- ⁹P. M. Levy and A. Fert, *Phys. Rev. B* **23**, 4667 (1981).
- ¹⁰B. L. Al'tshuler, A. G. Aronov, and A. Yu. Zyuzin, *Pis'ma Zh. Eksp. Teor. Fiz.* **38**, 128 (1983) [*JETP Lett.* **38**, 153 (1983)].
- ¹¹P. G. de Gennes, *Superconductivity of Metals and Alloys*, Benjamin, New York, 1966 (Russ. transl., Mir, Moscow, 1966).

Translated by Dave Parsons