RKKY interaction in metals with impurities

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The magnitude of the RKKY interaction in a metal with impurities is derived. The impurities are shown to cause a random phase shift of the RKKY interaction, but they do not cause an exponential decay of this interaction over distances on the order of the electron mean free path. This decay is found only for the interaction averaged over impurity configurations. However, it is meaningless to use such an average because of the pronounced dispersion of the interaction in the region in which the exponential factor is small.

The Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction between localized magnetic moments is known to fall off with increasing distance (r) between the magnetic moments, in proportion to r^{-3}. Furthermore, at $k_F r \gg 1$ the interaction is proportional to $\cos(2k_F r)$, i.e., is oscillatory (k_F is the Fermi momentum). There is a widely held opinion (Refs. 2 and 3, for example), dating back to a paper by de Gennes,[1] that in metals with impurities the magnitude of this interaction contains an additional factor $\exp(-r/l)$ and that it decays exponentially over a distance r greater than the mean free path (l) of the conduction electrons. In the present letter we show that this decay occurs only as a result of an averaging of the interaction over various impurity configurations, while the unaveraged RKKY interaction does not have an exponential factor. It contains only a random phase shift; i.e., it is given by

$$J(\mathbf{x},\mathbf{x}') \sim (k_F r)^{-3}\cos(2k_F r + \varphi), \quad r = |\mathbf{x}-\mathbf{x}'| \gg k_F^{-1} , \tag{1}$$

where the phase φ depends on $\mathbf{x}$, $\mathbf{x}'$, and the impurity configuration.

The magnitude of the RKKY interaction is determined by the susceptibility $\chi(\mathbf{x},\mathbf{x}')$ of the electron gas[4]:

$$J(\mathbf{x},\mathbf{x}') = I^2 \chi(\mathbf{x},\mathbf{x}')/g^2\mu_B^2 , \tag{2}$$

where g is the Landé factor, μ_B is the Bohr magneton, and I is the exchange integral, whose coordinate dependence we will be ignoring below. Expression (2) reflects the circumstance that the system loses its translational invariance in the presence of impurities, and the susceptibility $\chi(\mathbf{x},\mathbf{x}')$ is a function of both of the coordinates $\mathbf{x}$ and $\mathbf{x}'$. The susceptibility can be expressed in terms of the one-electron Green's functions G found by taking into account the scattering of electrons by impurities and in the temperature technique:

$$\chi(\mathbf{x},\mathbf{x}') = g^2\mu_B T \sum_\omega G_\omega^2(\mathbf{x},\mathbf{x}'), \qquad \omega = \pi T(2n+1) , \tag{3}$$

where T is the temperature.

In finding the Green's function we assume that the potential, $U(\mathbf{x})$, for the scattering by a given impurity distribution is a semiclassical potential. Taking this approach, we can find an explicit expression for the phase shift φ, although it is clear that expression (1) for the RKKY interaction holds even for a more general form of the random potential $U(\mathbf{x})$.

Semiclassical Green's functions for electrons were found in Refs. 5 and 6. They are equal to the sum of the contributions of the classical trajectories (Γ_i) of a particle with an energy ϵ in a given potential $U(\mathbf{x})$ connecting the points $\mathbf{x}$ and $\mathbf{x}'$. Among these trajectories there are some that connect these points directly, while there are also oscillatory trajectories, on which a particle undergoes one or several reflections from a turning point. The contribution from a straight trajectory Γ_0, whose length vanishes in the limit $\mathbf{x}\to\mathbf{x}'$, is (m is the mass of an electron)

$$G_\epsilon(\mathbf{x},\mathbf{x}') \sim \frac{1}{|\mathbf{x}-\mathbf{x}'|}\exp\left(\frac{i}{\hbar}S_0\{U(\mathbf{x})\}\right),$$
$$S_0 = \int_{\Gamma_0} ds\sqrt{2m[\epsilon - U(\mathbf{x})]} . \tag{4}$$

The arc length of the oscillatory trajectory remains finite in the limit $\mathbf{x}\to\mathbf{x}'$, and the magnitude of the action S_i undergoes pronounced fluctuations on this trajectory upon a change in the impurity configuration. We will therefore be ignoring the contribution of such oscillatory trajectories to the Green's function, which is determined entirely by expression (4). In the case of weak potential, $|U(\mathbf{x})| \ll \epsilon$, there are no oscillatory trajectories at all.

The temperature Green's function of the electrons in a metal, G_ω, is found through an analytic continuation $\epsilon = \mu + i\omega$ of the semiclassical Green's function in (4), where μ is the chemical potential. When the magnitude of the potential $U(\mathbf{x})$, for scattering by impurities, is small, $|U(\mathbf{x})| \ll \mu$, we can expand the action S_0 in (4) in $U(x)$. Ignoring the deviations of the trajectory Γ_0 from a rectilinear trajectory (the corresponding corrections are on the order of U^2), and substituting Green's function (4) into (3), we find, in first order in U,

$$\chi(\mathbf{x},\mathbf{x}') = \frac{N(0)}{16\pi r^3}\cos(2k_F r + \varphi), \quad \varphi = \frac{2}{\hbar v_F}\int_0^r ds\, U(\mathbf{x} + \mathbf{n}s), \tag{5}$$

where $\mathbf{n}$ is a unit vector connecting the points $\mathbf{x}$ and $\mathbf{x}'$, and v_F is the Fermi velocity. Expression (5) leads to result (1) for the magnitude of the RKKY interaction. Averaging it over impurity configurations, we find

$$\langle J(\mathbf{x},\mathbf{x}')\rangle \sim (k_F r)^{-3}\cos(2k_F r)\exp(-r/l), \qquad r \gg \xi. \tag{6}$$

Here ξ is the correlation radius of the random potential $U(\mathbf{x})$, whose correlation function $g(\mathbf{x}-\mathbf{x}') = \langle U(\mathbf{x})U(\mathbf{x}')\rangle$ determines the mean free path:

$$l^{-1} = \frac{4}{\hbar^2 v_F^2}\int_{-\infty}^{+\infty} dr g(r), \qquad l \gg \xi. \tag{7}$$

In the case $\xi \gg l$, we would generally have to take into account the non-Gaussian nature of the fluctuations of the potential $U(\mathbf{x})$ in order to find l. In the case $l \gg \xi$, and also in the case of an ordinary diffusive motion of electrons in the field of the random Gaussian potential $U(x)$, we find the following expression for the dispersion of the RKKY interaction:

$$\langle J^2(\mathbf{x},\mathbf{x}')\rangle - \langle J(\mathbf{x},\mathbf{x}')\rangle^2 = 2sh^2(r/l)\langle J(\mathbf{x},\mathbf{x}')\rangle^2. \tag{8}$$

We see from (8) that in the region $r \gg l$, where the exponential factor in (6) is small, the fluctuations $J(\mathbf{x},\mathbf{x}')$ become extremely large, and a description of the RKKY interaction exclusively in terms of its mean value becomes meaningless.

In summary, in metals containing static impurities the RKKY interaction falls off over distance only as r^{-3}, but impurities do cause the phase of the oscillations of the RKKY interaction to become random. This effect intensifies the tendency toward the formation of an ordering of the spin-glass type in metals with a random arrangement of localized magnetic moments. Even in the case of a regular arrangement of magnetic moments, the presence of nonmagnetic impurities makes the RKKY interaction a random interaction and thus raises the possibility of a spin-glass ordering.

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