Dynamics of the Bloch lines at high velocities

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The structure of a Bloch line has been determined for the first time up to its critical velocity. New solutions of the Slonczewski equations, which describe the steady motion of a family of Bloch lines, have been found.

1. The Slonczewski equations are widely used in high-Q ferromagnets[1] $K_u/2\pi M^2 \gg 1$ (K_u is the uniaxial anisotropy constant, and M is the magnetization of a unit volume) to describe solitary spin waves which propagate along a domain wall and which are called Bloch lines in the literature. Many studies of these equations have dealt with the region of linear dynamics of solitary waves in which the velocities of these waves are far from the phase velocity of the near-wall magnons in the linear part of their spectrum $S = \gamma\sqrt{8\pi A}$, where γ is the gyromagnetic ratio, and A is the constant of the inhomogeneous volume. The critical velocity of a single Bloch line was discussed in Ref. 2, in which it was pointed out that there are two critical velocities $V_\pm = S|1 \pm b|$ (where $b^2 = H'\Delta/4\pi M$, $\Delta = \sqrt{A/K_u}$, and H' is the bias-field gradient) at which the asymptotic approach of the magnetic system to the equilibrium state changes. Analysis of the asymptotic behavior does not, however, answer the question as to whether there are separatrix solutions and what their topological structure might be. To analyze the steady-state solutions of the Slonczewski equations, we use a method of qualitative and numerical analyses which was described in Ref. 3 and which makes it possible to answer these questions. For simplicity, we will consider an untwisted Bloch domain wall and take the demagnetization fields into account in the Winter approximation. In this case we can write the basic equations in the form

$$\begin{cases} -uq_\xi = \dfrac{1}{2}\sin 2\varphi - \varphi_{\xi\xi}, \\ -u\varphi_\xi = q_{\xi\xi} - b^2 q, \qquad \xi = x - ut, \end{cases} \tag{1}$$

where u is the velocity which is normalized to S, $q(\xi)$ is the divergence of the domain-wall center from the equilibrium position (the magnitude of the divergence is measured in terms of the domain-wall thickness Δ), $\varphi(\xi)$ is the angle at which the magnetization emerges from the domain-wall plane, and x is the coordinate which is directed along the domain wall and which is normalized to the Bloch-line thickness ($\Delta_L = \sqrt{A/2\pi M^2}$) for $u = 0$. In Eq. (1) we have dropped the terms which describe the dissipation and influx of energy to the magnetic system, since we are studying the inertial motion of the Bloch lines.

2. The solitary wave which describes the moving Bloch line in four-dimensional phase space $(\varphi, \varphi', q, q')$ corresponds to the separatrix curve which links two adjacent singularities a_n and a_{n+1}, where $n = 0, \pm 1, \pm 2, \ldots$, $a_n = (n\pi, 0, 0, 0)$. Specifically, for $u = 0$ the solution is familiar:

$$q = 0, \qquad \varphi = \pi n \pm 2\arctan\exp(\xi). \tag{2}$$

There are no other separatrix solutions for $u = 0$. Linearizing the system of equations (1) near the singularity $a(\xi) = a_n + \delta a$, we can analyze the behavior of the equations in the limit $\xi \to \pm\infty$. An asymptotic analysis (see Ref. 2) shows that for $u < u_- = |1 - b|$ the singularities a_n are saddle-saddle singularities. Taking the first integral in (1) into account, we can thus construct separatrix solutions in the entire region $u < u_-$ by using the method suggested in Ref. 3. Figure 1(a) shows a solution

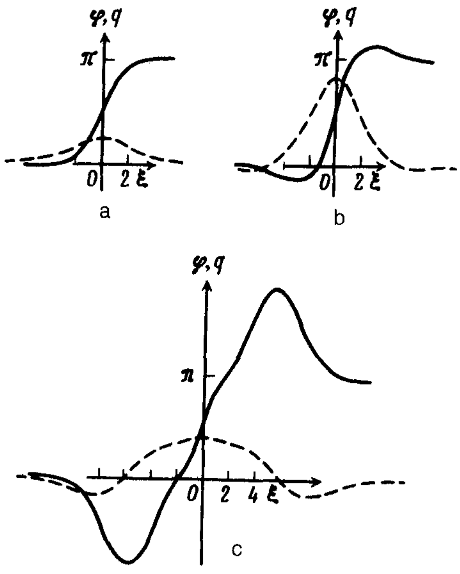

FIG. 1. The $\varphi(\xi)$ curve (solid curve) and the $q(\xi)$ curve (dashed curve) describe the structure of a moving solitary wave.

of (1) for $b = 0.5$ and $u = 0.4$, which connects the singularities a_0 and a_1. We wish to point out the characteristic bend in the "tails" of the $\varphi(\xi)$ curve which appears when the Bloch line is in motion ($u \neq 0$) and which accounts for the fact that the oscillation amplitude of this angle is slightly higher than π. An increase in the velocity causes both the domain-wall curvature $q(\xi)$ and the oscillation amplitude $\varphi(\xi)$ of the wave to increase.

3. In the region $u_- < u < u_+ = 1 + b$ the characteristic indices of the linearized system of equations (1) are complex conjugates and the singularities a_n are saddle-focus singularities. The monotonic input and output of the solution from the singularity in this case begins to oscillate at $u < u_-$ in the region $u_- < u < u_+$. A numerical integration of (1) for $u > u_-$ has shown that a very simple separatrix solution, which links the adjacent singularities, does not apply to the entire velocity range, $u_- < u < u_+$, and is bounded above by $u_c < u_+$, where u_c is the critical velocity of a single Bloch line. Specifically, for $b = 0.5$ the critical velocity is $u_c \sim 1.05$. An example of a separatrix solution corresponding to an isolated Bloch line at $u = 1.0$ and $b = 0.5$ is shown in Fig. 1(b). A comparison with Fig. 1(a) shows that a further increase in the velocity increases the domain-wall curvature and the divergence of the angle $\varphi(\xi)$ in the wave. As we go through the velocity u_-, only the "tails" of the solution oscillate, while its topological structure remains largely unaffected.

4. The noncontinuability of the solution of the type in question in the region $u > u_c$ stems from the fact that it combines with another solution which breaks up into five Bloch lines in the limit $u \to 0$. A solution of this type, which describes a family of Bloch lines in motion for $u = 0.4$ and $b = 0.5$ is shown in Fig. 1(c). In a similar manner we found that solutions which correspond in the limit $u \to 0$ to three and seven Bloch lines, respectively, are also combined. A combining of solutions of this sort occurs at $u = u_{c'} > u_c$. These separatrix solutions give rise to single-parameter families (u is a parameter) which differ in their topological structure and which have their critical velocities, $u_c(b)$. Since at $u = 0$ we have a unique separatrix solution of (2) to within the sign, the solution of each family decomposes, in the limit $u \to 0$, into a series of solutions of the form (2), which are spatially separated by an arbitrarily large distance from each other.

5. The energy of the steadily moving Bloch lines, which are described by Eqs. (1), is given by

$$E(u) = \int_{-\infty}^{+\infty} [(q_\xi)^2 + (\varphi_\xi)^2] d\xi \; ^{1)} . \qquad (3)$$

Since the structure of the solution is known, we can easily find the function $E(u)$. Figure 2 shows this function for the two solutions considered by us. It is clear that in the limit $u \to 0$ the energy $E \to 2k$, where k is the number of Bloch lines corresponding to the chosen solution. With increasing velocity, the energy of the solitary wave increases, reaching a maximum value in the second solution at $u \sim 0.8u_c$. Upon reaching the critical velocity, the energy of a single Bloch line becomes equal to the energy of the family consisting of five lines. We see that the mass $m = d^2E/du^2$ of an isolated Bloch line increases without bound as $u \to u_c$. The mass is negative in the second solution at $u \sim 0.8u_c$, indicating that this solution is unstable at high velocities. Figure

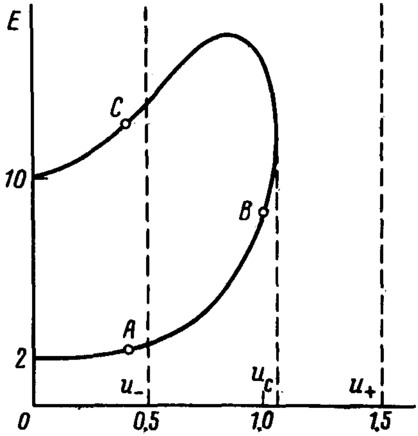

FIG. 2. The velocity dependence of the energy of the moving Bloch lines. Points A, B, and C on the plot correspond to the solutions in Figs. 1(a)–1(c), respectively.

2 also shows that bifurcation of the equilibrium state on going through the critical velocity, $u = u_-$, has no effect on the function $E = E(u)$, leaving it a monotonic function. A dynamical degeneracy of the two solutions, which differ topologically at low velocities, indicates that a pair of Bloch lines can, in principle, be generated inside the volume as the critical velocity is reached and subsequently lowered.

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[1)]In writing the expression for the energy of a solitary wave (3), we took into account that the system of equations (1) has a first integral $\sin^2\varphi + b^2 q^2 = (q_\xi)^2 + (\varphi_\xi)^2$.

[1]A. Malozemov and J. C. Slonczewski, Domennye stenki v materialakh s tsilindricheskimi magnitnymi domenami (Domain Walls in Materials with Magnetic Bubbles), Mir, Moscow, 1982.
[2]A. K. Zvezdin and A. F. Popkov, Pis'ma Zh. Eksp. Teor. Fiz. **41**, 90 (1985) [JETP Lett. **41**, 107 (1985)].
[3]V. M. Eleonskiĭ, N. N. Kirova, and N. E. Kulagin, Zh. Eksp. Teor. Fiz. **75**, 2210 (1978) [Sov. Phys. JETP **48**, 1113 (1978)].

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